Name: ______________________________ Date: ____________________

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

Things to remember before you get started:

1. Write as a single fraction.

a. $\frac{2}{5}+\frac{3}{7}$

$= \frac{14}{35} + \frac{15}{35} = \frac{29}{35}$

b. $\frac{7}{x}+\frac{2x}{x+1}$ $\quad \frac{x+1}{x+1}$ $\quad \frac{x}{x}$

$= \frac{7(x+1)+2x(x)}{x(x+1)}$

$= \frac{2x^2+7x+7}{x^2+x}$

c. $\sec x+\frac{\cos x}{\sin x}$ $\quad \frac{\sin x}{\sin x}$ $\quad \frac{\cos x}{\cos x}$

$= \frac{1}{\cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin x + \cos^2 x}{\sin x \cos x}$

2. Simplify.

a. $\frac{1+\frac{1}{x}}{\frac{y}{x}}$ $\quad \frac{x}{x}$

$= \frac{\left(\frac{x+1}{x}\right)}{\frac{y}{x}}$

$= \left(\frac{x+1}{x}\right)\left(\frac{x}{y}\right)$

$= \frac{x+1}{y}$

b. $\sin x+\tan x$

$= \sin x + \frac{\sin x}{\cos x}$

$= \frac{\sin x\cos x + \sin x}{\cos x}$

$= \frac{\sin x(\cos x + 1)}{\cos x}$

c. $\frac{\cos x+\tan x}{\cot x+\sec x}$

$= \frac{\cos x + \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{1}{\cos x}}$

$= \frac{\left(\frac{\cos^2 x + \sin x}{\cos x}\right)}{\left(\frac{\cos^2 x + \sin x}{\sin x\cos x}\right)}$

$\left(\frac{\cos^2 x + \sin x}{\cos x}\right)\left(\frac{\sin x\cos x}{\cos^2 x + \sin x}\right)$

$= \sin x$

3. Factor.

a. $2a+14ab$

$= 2a(1+7b)$

b. $\sin x\cos x+\cos x$

$= \cos x(\sin x + 1)$

c. $\tan^2 x-16$

$y = \tan x$

$y^2 - 16$

$= (y+4)(y-4)$

$= (\tan x + 4)(\tan x - 4)$

d. $\tan^2 x+5\tan x-6$

$y = \tan x$

$y^2 + 5y - 6$

$= (y-1)(y+6)$

$= (\tan x - 1)(\tan x + 6)$

4. State the non-permissible values (restrictions).

a. $\frac{3x}{x}-\frac{7}{2x^2-4x+2}$

NPV: ↑ $x \neq 0$

$2(x^2-2x+1) \neq 0$

$2(x-1)^2 \neq 0$

$x-1 \neq 0$

$x \neq 1$

b. $\csc x$

$= \frac{1}{\sin x}$

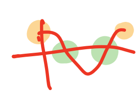
NPV: $\sin x \neq 0$

$x \neq 0, \pi, 2\pi$

c. $\frac{\sin x\tan x}{\cos x-1}$

① Denominator = 0

$\cos x - 1 \neq 0$



$\cos x \neq 1$

$x \neq 2\pi, 0$

② Expression undefined

$\tan x \neq$ DNE

$\frac{\sin x}{\cos x} \neq$ DNE

$\cos x \neq 0$

$x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

Assignment p. 314 # 1 – 18, C2, C3 Quiz Next Day!

Example Prove. State any non – permissible values.

a. $\tan x = \dfrac{1 - \cos 2x}{\sin 2x}$

$= \dfrac{1 - (2\cos^2 x - 1)}{2\sin x \cos x}$

$= \dfrac{1 - 2\cos^2 x + 1}{2\sin x \cos x}$

$= \dfrac{2 - 2\cos^2 x}{2\sin x \cos x}$

$= \dfrac{2(1 - \cos^2 x)}{2\sin x \cos x} \leftarrow \sin^2 x = 1 - \cos^2 x$

$= \dfrac{\sin^2 x}{\sin x \cos x}$

$= \dfrac{\sin x}{\cos x}$

$= \tan x$

b. $\dfrac{1}{1 + \sin x} = \dfrac{\sec x - \sin x \sec x}{\cos x}$

$= \dfrac{\left(\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}\right)}{\cos x}$

$= \left(\dfrac{1 - \sin x}{\cos x}\right)\left(\dfrac{1}{\cos x}\right)$

$= \boxed{\dfrac{1 - \sin x}{\cos^2 x}}$

$= \dfrac{1 - \sin x}{1 - \sin^2 x} \leftarrow 1 - y^2 = (1+y)(1-y)$

$= \dfrac{1 - \sin x}{(1 + \sin x)(1 - \sin x)}$

$= \dfrac{1}{1 + \sin x}$

c. $\dfrac{\sin 2x - \cos x}{4\sin^2 x - 1} = \dfrac{\sin^2 x \cos x + \cos^3 x}{2\sin x + 1}$

$= \dfrac{2\sin x \cos x - \cos x}{4\sin^2 x - 1}$

$= \dfrac{\cos x(2\sin x - 1)}{4\sin^2 x - 1}$

$= \dfrac{\cos x(2\sin x - 1)}{(2\sin x + 1)(2\sin x - 1)}$

$= \dfrac{\cos x}{2\sin x + 1}$

$= \dfrac{\cos x(\sin^2 x + \cos^2 x)}{2\sin x + 1}$

$= \dfrac{\cos x}{2\sin x + 1} \times \dfrac{2\sin x - 1}{2\sin x - 1}$

$= \dfrac{\cos x(2\sin x - 1)}{4\sin^2 x - 2\sin x + 2\sin x - 1}$

$= \dfrac{\cos x(2\sin x - 1)}{4\sin^2 x - 1}$

d. $\dfrac{1+\sin\theta}{1+\sin\theta} \times \dfrac{1}{1 - \sin\theta} = \dfrac{1 + \sin\theta}{\cos^2\theta} \times \dfrac{1 - \sin\theta}{1 - \sin\theta}$

$= \dfrac{1 + \sin\theta}{1 - \sin\theta + \sin\theta - \sin^2\theta}$

$= \dfrac{1 + \sin\theta}{1 - \sin^2\theta}$

$= \dfrac{1 + \sin\theta}{\cos^2\theta}$

$= \dfrac{1 - \sin\theta + \sin\theta - \sin^2\theta}{\cos^2\theta(1 - \sin\theta)}$

$= \dfrac{1 - \sin^2\theta}{\cos^2\theta(1 - \sin\theta)}$

$= \dfrac{\cos^2\theta}{\cos^2\theta(1 - \sin\theta)}$

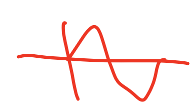
$= \dfrac{1}{1 - \sin\theta}$

NPVs.

$1 - \sin\theta \neq 0$

$\sin\theta \neq 1$

$\theta \neq \frac{\pi}{2}$



$\cos^2\theta \neq 0$

$\cos\theta \neq 0$

$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$