Name: ________________________ Date: __________________

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

More Questions – Solutions

1. Prove. State any non – permissible values.

a. $\dfrac{1-\cos\alpha}{\sin\alpha}=\dfrac{\sin\alpha}{1+\cos\alpha}$

$\dfrac{1-\cos\alpha}{\sin\alpha}$	$\dfrac{\sin\alpha}{1+\cos\alpha}$
	$=\dfrac{\sin\alpha}{1+\cos\alpha}\times\dfrac{1-\cos\alpha}{1-\cos\alpha}$
	$=\dfrac{\sin\alpha\,(1-\cos\alpha)}{1-\cos^2\alpha}$
	$=\dfrac{\sin\alpha\,(1-\cos\alpha)}{\sin^2\alpha}$
	$=\dfrac{1-\cos\alpha}{\sin\alpha}$

b. $\dfrac{1}{1-\sin\theta}=\dfrac{1+\sin\theta}{\cos^2\theta}$

$\dfrac{1}{1-\sin\theta}$	$\dfrac{1+\sin\theta}{\cos^2\theta}$
$=\dfrac{1}{1-\sin\theta}\times\dfrac{1+\sin\alpha}{1+\sin\alpha}$	
$=\dfrac{1+\sin\alpha}{1-\sin^2\theta}$	
$=\dfrac{1+\sin\alpha}{\cos^2\theta}$	

c. $\dfrac{\sin x + \tan x}{\cos x + 1} = \dfrac{\sec x}{\csc x}$

$\dfrac{\sin x + \tan x}{\cos x + 1}$	$\dfrac{\sec x}{\csc x}$
$= \dfrac{\sin x + \tan x}{\cos x + 1} \times \dfrac{1 - \cos x}{1 - \cos x}$ $= \dfrac{(\sin x + \tan x)(1 - \cos x)}{1 - \cos^2 x}$ $= \dfrac{\sin x - \sin x \cos x + \tan x - \cos x \tan x}{\sin^2 x}$ $= \dfrac{\sin x - \sin x \cos x + \tan x - \cos x \times \frac{\sin x}{\cos x}}{\sin^2 x}$ $= \dfrac{\sin x - \sin x \cos x + \tan x - \sin x}{\sin^2 x}$ $= \dfrac{-\sin x \cos x + \tan x}{\sin^2 x}$ $= \dfrac{\frac{-\sin x \cos^2 x}{\cos x} + \frac{\sin x}{\cos x}}{\sin^2 x}$ $= \dfrac{\sin x\,(1 - \cos^2 x)}{\cos x} \Big/ \sin^2 x$ $= \dfrac{\sin x\,(\sin^2 x)}{\cos x} \times \dfrac{1}{\sin^2 x}$ $= \dfrac{\sin x}{\cos x}$	$= \dfrac{1}{\cos x} \Big/ \dfrac{1}{\sin x}$ $= \dfrac{1}{\cos x} \times \sin x$ $= \dfrac{\sin x}{\cos x}$

d. $\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$

$\frac{\cos x}{1-\sin x}$	$\frac{1+\sin x}{\cos x}$
$= \frac{\cos x}{1-\sin x} \times \frac{1+\sin x}{1+\sin x}$ $= \frac{\cos x\,(1+\sin x)}{1-\sin^2 x}$ $= \frac{\cos x\,(1+\sin x)}{\cos^2 x}$ $= \frac{1+\sin x}{\cos x}$	

e. $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{2}{\cos\theta}$

$\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta}$	$\frac{2}{\cos\theta}$
$= \left(\frac{\cos\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}\right) + \left(\frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}\right)$ $= \left(\frac{\cos\theta\,(1+\sin\theta)}{1-\sin^2\theta}\right) + \left(\frac{\cos\theta\,(1-\sin\theta)}{1-\sin^2\theta}\right)$ $= \left(\frac{\cos\theta + \sin\theta\cos\theta}{1-\sin^2\theta}\right) + \left(\frac{\cos\theta - \sin\theta\cos\theta}{1-\sin^2\theta}\right)$ $= \frac{\cos\theta + \sin\theta\cos\theta + \cos\theta - \sin\theta\cos\theta}{1-\sin^2\theta}$ $= \frac{2\cos\theta}{1-\sin^2\theta}$ $= \frac{2\cos\theta}{\cos^2\theta}$ $= \frac{2}{\cos\theta}$	