

Name: _____

Date: _____

Learning Goal 6.2

Solving equations, identifying any non-permissible values and extraneous roots.

1. Solve the following rational equations. State any non – permissible values and/or extraneous roots.

a.
$$\frac{-3}{x+2} = \frac{2x}{x-3}$$

b.
$$\frac{x+2}{x-5} = \frac{x}{x-1}$$

Non-Permissible Values:

$$\begin{array}{ll} x+2 \neq 0 & x-3 \neq 0 \\ x \neq -2 & x \neq 3 \end{array}$$

Non-Permissible Values:

$$\begin{array}{ll} x-5 \neq 0 & x-1 \neq 0 \\ x \neq 5 & x \neq 1 \end{array}$$

$$\begin{aligned} \frac{-3(x-3)}{(x+2)(x-3)} &= \frac{2x(x+2)}{(x-3)(x+2)} \\ -3(x-3) &= 2x(x+2) \\ -3x+9 &= 2x^2+4x \\ 0 &= 2x^2+7x-9 \\ 0 &= 2x^2-2x+9x-9 \\ 0 &= 2x(x-1)+9(x-1) \\ 0 &= (x-1)(2x+9) \\ x-1 &= 0 & 2x+9 &= 0 \\ x &= 1 & 2x &= -9 \\ & & x &= -\frac{9}{2} \end{aligned}$$

$$\begin{aligned} \frac{(x+2)(x-1)}{(x-5)(x-1)} &= \frac{x(x-5)}{(x-1)(x-5)} \\ (x+2)(x-1) &= x(x-5) \\ x^2+x-2 &= x^2-5x \\ 6x-2 &= 0 \\ 2(3x-1) &= 0 \\ 3x-1 &= 0 \\ 3x &= 1 \\ x &= \frac{1}{3} \end{aligned}$$

Check:

$$\begin{aligned} \frac{-3}{(1)+2} &= \frac{2(1)}{(1)-3} & \frac{-3}{(-\frac{9}{2})+2} &= \frac{2(-\frac{9}{2})}{(-\frac{9}{2})-3} \\ \frac{-3}{3} &= \frac{2}{-2} & \frac{-3}{-\frac{9}{2}} &= \frac{-9}{-\frac{9}{2}} \\ -1 &= -1 & \frac{-3}{-\frac{9}{2}} &= \frac{-9}{-\frac{9}{2}} \\ & & \frac{-6}{-5} &= \frac{-18}{-15} \\ & & \frac{-6}{-5} &= \frac{-6}{-5} \end{aligned}$$

Check

$$\begin{aligned} \frac{(\frac{1}{3})+2}{(\frac{1}{3})-5} &= \frac{(\frac{1}{3})}{(\frac{1}{3})-1} \\ \frac{(\frac{1}{3})+\frac{6}{3}}{(\frac{1}{3})-\frac{15}{3}} &= \frac{(\frac{1}{3})}{(\frac{1}{3})-\frac{3}{3}} \\ \frac{7/3}{-14/3} &= \frac{1/3}{-2/3} \\ \frac{7}{-14} &= \frac{1}{-2} \\ \frac{1}{-2} &= \frac{1}{-2} \end{aligned}$$

$$c. \frac{3x + 1}{x^2 - 1} = \frac{-x}{x + 1}$$

Non-Permissible

Values:

$$\begin{aligned} x^2 - 1 &\neq 0 & x + 1 &\neq 0 \\ (x - 1)(x + 1) &\neq 0 & x &\neq -1 \\ x &\neq \pm 1 \\ \frac{3x + 1}{(x - 1)(x + 1)} &= \frac{-x(x - 1)}{(x + 1)(x - 1)} \\ 3x + 1 &= -x^2 + x \\ x^2 + 2x + 1 &= 0 \\ (x + 1)^2 &= 0 \\ x + 1 &= 0 \\ x &= -1 \end{aligned}$$

No Real Solutions

$$d. \frac{7x - 15}{x^2 - 9} = \frac{x - 2}{x - 3}$$

Non-Permissible Values:

$$\begin{aligned} x^2 - 9 &\neq 0 & x - 3 &\neq 0 \\ (x - 3)(x + 3) &\neq 0 & x &\neq 3 \\ x &\neq \pm 3 \\ \frac{7x - 15}{(x - 3)(x + 3)} &= \frac{(x - 2)(x + 3)}{(x - 3)(x + 3)} \\ 7x - 15 &= (x - 2)(x + 3) \\ 7x - 15 &= x^2 + x - 6 \\ 0 &= x^2 - 6x + 9 \\ 0 &= (x - 3)^2 \\ 0 &= x - 3 \\ 3 &= x \end{aligned}$$

No Real Solutions

$$e. \frac{9}{y-3} - \frac{4}{y-6} = \frac{18}{y^2 - 9y + 18}$$

Non-Permissible Values:

$$\begin{array}{l} y-3 \neq 0 \quad y-6 \neq 0 \quad y^2 - 9y + 18 \neq 0 \\ y \neq 3 \quad y \neq 6 \quad (y-3)(y-6) \neq 0 \\ y-3 \neq 0 \quad y-6 \neq 0 \\ y \neq 3 \quad y \neq 6 \end{array}$$

$$\begin{aligned} \frac{9(y-6)}{(y-3)(y-6)} - \frac{4(y-3)}{(y-6)(y-3)} &= \frac{18}{(y-3)(y-6)} \\ 9(y-6) - 4(y-3) &= 18 \\ 9y - 54 - 4y + 12 &= 18 \\ 5y - 42 &= 18 \\ 5y &= 60 \\ y &= 12 \end{aligned}$$

Check:

$$\begin{aligned} \frac{9}{(12)-3} - \frac{4}{(12)-6} &= \frac{18}{(12)^2 - 9(12) + 18} \\ \frac{9}{9} - \frac{4}{6} &= \frac{18}{144 - 108 + 18} \\ \frac{6}{6} - \frac{4}{6} &= \frac{18}{54} \\ \frac{2}{6} &= \frac{2}{6} \end{aligned}$$

$$f. \frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{x^2 - x - 6}$$

Non-Permissible Values:

$$\begin{array}{l} x+2 \neq 0 \quad x-3 \neq 0 \quad x^2 - x - 6 \neq 0 \\ x \neq -2 \quad x \neq 3 \quad (x-3)(x+2) \neq 0 \\ x-3 \neq 0 \quad x+2 \neq 0 \\ x \neq 3 \quad x \neq -2 \end{array}$$

$$\begin{aligned} \frac{3x(x-3)}{(x+2)(x-3)} - \frac{5(x+2)}{(x-3)(x+2)} &= \frac{-25}{(x-3)(x+2)} \\ 3x(x-3) - 5(x+2) &= -25 \\ 3x^2 - 9x - 5x - 10 &= -25 \\ 3x^2 - 14x - 10 &= -25 \\ 3x^2 - 14x + 15 &= 0 \\ 3x^2 - 9x - 5x + 15 &= 0 \\ 3x(x-3) - 5(x-3) &= 0 \\ (x-3)(3x-5) &= 0 \end{aligned}$$

$$\begin{array}{l} x-3 = 0 \quad 3x-5 = 0 \\ x = 3 \quad 3x = 5 \\ \text{Nope!} \quad x = \frac{5}{3} \end{array}$$

Check:

$$\begin{aligned} \frac{3\left(\frac{5}{3}\right)}{\left(\frac{5}{3}\right)+2} - \frac{5}{\left(\frac{5}{3}\right)-3} &= \frac{-25}{\left(\frac{5}{3}\right)^2 - \left(\frac{5}{3}\right) - 6} \\ \frac{5}{5} - \frac{5}{5} &= \frac{-25}{-25} \\ \frac{\left(\frac{5}{3}\right) + \frac{6}{3}}{5} - \frac{\left(\frac{5}{3}\right) - \frac{9}{3}}{5} &= \frac{\frac{25}{9} - \left(\frac{5}{3}\right) - 6}{-25} \\ \frac{11}{3} - \frac{-4}{3} &= \frac{\frac{25}{9} - \left(\frac{15}{9}\right) - \frac{54}{9}}{-25} \\ \frac{15}{11} + \frac{15}{4} &= \frac{-25}{-44} \\ \frac{60}{44} + \frac{165}{44} &= \frac{225}{44} \\ \frac{225}{44} &= \frac{225}{44} \end{aligned}$$

2. Stella takes 4 hours to paint a room. It takes Jose 3 hours to paint the same area. How long will the paint job take if they work together?

Stella:

4 hours for one room
1 hour for $\frac{1}{4}$ of a room

Jose:

3 hours for one room
1 hour for $\frac{1}{3}$ of a room

In one hour they can paint $\frac{7}{12}$ of the room together.

$$\begin{aligned}\frac{1}{4} + \frac{1}{3} &= \frac{3}{12} + \frac{4}{12} \\ &= \frac{7}{12}\end{aligned}$$

So the whole room will take $\frac{12}{7}$, or ≈ 1.7 hours for them both to paint.

$$\frac{7}{12}x = 1$$

$$7x = 12$$

$$x = \frac{12}{7}$$