

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Learning Goal 8.1**Solving exponential and logarithmic equations with same base and with different bases, including base  $e$ .**More Questions - Solutions**

Power Law	Product Law	Quotient Law	Change of Base
$\log_b x^y = y \log_b x$	$\log_b(xy) = \log_b x + \log_b y$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_b x = \frac{\log_a x}{\log_a b}$

1. Solve for  $x$ . Round your answers to the nearest hundredth.

a.  $2^z = 2500$

$$\log(2^z) = \log(2500)$$

$$z \log(2) = \log(2500)$$

$$z = \frac{\log(2500)}{\log(2)}$$

$$z \approx 11.29$$

b.  $5^{x-3} = 1700$

$$\log(5^{x-3}) = \log(1700)$$

$$(x-3) \log(5) = \log(1700)$$

$$x-3 = \frac{\log(1700)}{\log(5)}$$

$$x = \frac{\log(1700)}{\log(5)} + 3$$

$$x \approx 7.62$$

c.  $8(3^{2x}) = 568$

$$3^{2x} = 71$$

$$\log(3^{2x}) = \log(71)$$

$$2x \log(3) = \log(71)$$

$$2x = \frac{\log(71)}{\log(3)}$$

$$x = \frac{\log(71)}{2 \log(3)}$$

$$x \approx 1.94$$

d.  $6^{3x+1} = 8^{x+3}$

$$\log(6^{3x+1}) = \log(8^{x+3})$$

$$(3x+1) \log(6) = (x+3) \log(8)$$

$$3x \log 6 + \log 6 = x \log 8 + 3 \log 8$$

$$3x \log 6 + \log 6 = x \log 8 + \log 512$$

$$3x \log 6 - x \log 8 = \log 512 - \log 6$$

$$x(3 \log 6 - \log 8) = \log\left(\frac{512}{6}\right)$$

$$x(\log 216 - \log 8) = \log\left(\frac{256}{3}\right)$$

$$x \log\left(\frac{216}{8}\right) = \log\left(\frac{256}{3}\right)$$

$$x \log(27) = \log\left(\frac{256}{3}\right)$$

$$x = \frac{\log\left(\frac{256}{3}\right)}{\log(27)}$$

$$x \approx 1.35$$

## Day 1

$$e. \quad 4(7^{x+2}) = 9^{2x-3}$$

$$\log(4(7^{x+2})) = \log(9^{2x-3})$$

$$\log(4) + \log(7^{x+2}) = \log(9^{2x-3})$$

$$\log(4) + (x+2)\log(7) = (2x-3)\log(9)$$

$$\log(4) + x\log(7) + 2\log(7) = 2x\log(9) - 3\log(9)$$

$$\log(4) + x\log(7) + \log(49) = x\log(81) - \log(729)$$

$$x\log(7) - x\log(81) = -\log(729) - \log(4) - \log(49)$$

$$x(\log 7 - \log 81) = \log\left(\frac{1}{729}\right) + \log\left(\frac{1}{4}\right) + \log\left(\frac{1}{49}\right)$$

$$x\log\left(\frac{7}{81}\right) = \log\left(\frac{1}{729} \times \frac{1}{4} \times \frac{1}{49}\right)$$

$$x\log\left(\frac{7}{81}\right) = \log\left(\frac{1}{142884}\right)$$

$$x = \frac{\log\left(\frac{1}{142884}\right)}{\log\left(\frac{7}{81}\right)}$$

$$x \approx -4.85$$

2. Find the half-life of an isotope if 10 grams of a 150 gram sample remains after 21.9 days.

$$w(t) = w_0 \times \left(\frac{1}{2}\right)^{\frac{t}{t_H}}$$

$$10 = 150 \times \left(\frac{1}{2}\right)^{\frac{21.9}{t_H}}$$

$$\frac{10}{150} = \left(\frac{1}{2}\right)^{\frac{21.9}{t_H}}$$

$$\frac{1}{15} = \left(\frac{1}{2}\right)^{\frac{21.9}{t_H}}$$

$$\log\left(\frac{1}{15}\right) = \log\left(\left(\frac{1}{2}\right)^{\frac{21.9}{t_H}}\right)$$

$$\log\left(\frac{1}{15}\right) = \left(\frac{21.9}{t_H}\right)\log\left(\frac{1}{2}\right)$$

$$t_H \log\left(\frac{1}{15}\right) = 21.9 \log\left(\frac{1}{2}\right)$$

$$t_H = \frac{21.9 \log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{15}\right)}$$

$$t_H \approx 5.60$$

The half-life of the isotope is approximately 5.6 days.