

Name: _____

Date: _____

Learning Goal 9.2

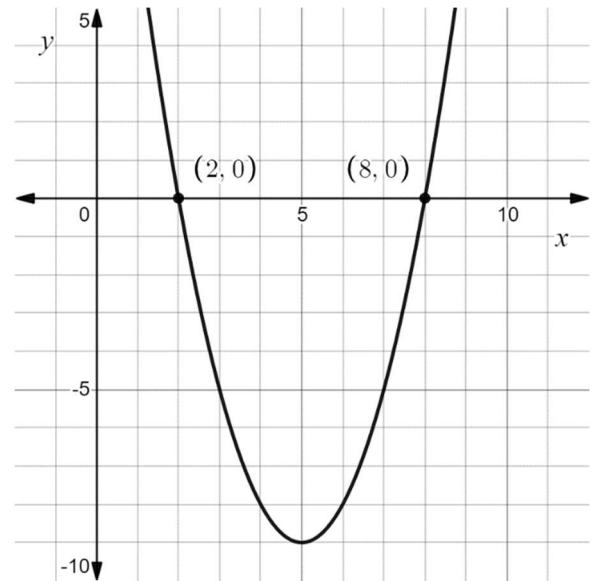
Solving quadratic inequalities.

Solve the following inequalities. Use each method at least once.

1. $x^2 - 10x + 16 > 0$

Method 1 Graphing

$\{x \mid x < 2, 8 < x, x \in \mathbb{R}\}$

**Method 2 Roots and Test Points**

$$\begin{aligned}
 x^2 - 10x + 16 &= 0 \\
 (x - 2)(x - 8) &= 0 \\
 x - 2 = 0 &\quad \text{or} \quad x - 8 = 0 \\
 x = 2 &\quad \quad \quad x = 8
 \end{aligned}$$



Interval	$x < 2$	$2 < x < 8$	$8 < x$
Test Pt.	0	5	10
Check	$(0)^2 - 10(0) + 16$ $= 0 - 0 + 16$ $= 16$	$(5)^2 - 10(5) + 16$ $= 25 - 50 + 16$ $= -9$	$(10)^2 - 10(10) + 16$ $= 100 - 100 + 16$ $= 16$
So?	$16 > 0$	$-9 < 0$	$16 > 0$

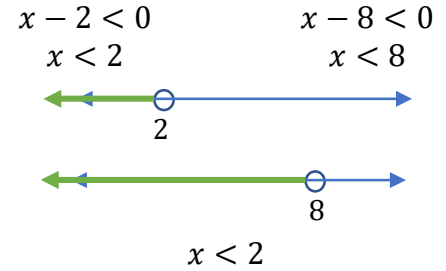
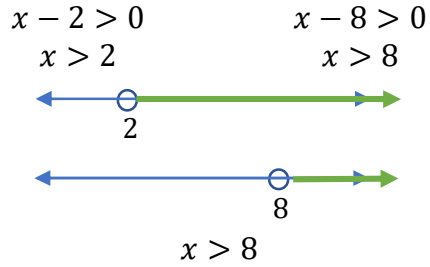
$\{x \mid x < 2, 8 < x, x \in \mathbb{R}\}$

Method 3 Case Analysis

$$x^2 - 10x + 16 > 0$$

$$(x - 2)(x - 8) > 0$$

So either both factors have to be positive or both have to be negative.

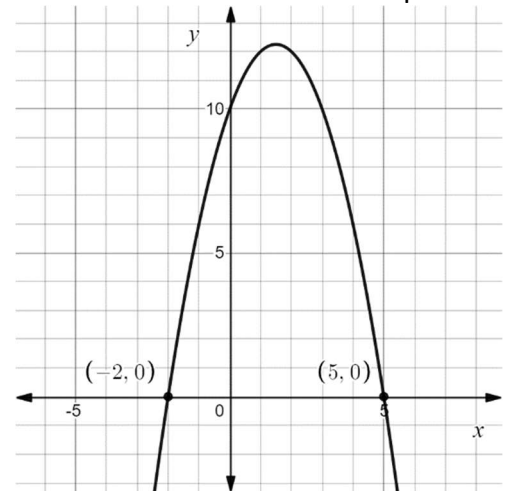


$$\{x \mid x < 2, 8 < x, x \in \mathbb{R}\}$$

2. $-x^2 + 3x + 10 \leq 0$

Method 1 Graphing

$\{x \mid x \leq -2, 5 \leq x, x \in \mathbb{R}\}$



Method 2 Roots and Test Points

$$\begin{aligned} -x^2 + 3x + 10 &= 0 \\ -(x^2 - 3x - 10) &= 0 \\ -(x - 5)(x + 2) &= 0 \\ x - 5 = 0 \quad \text{or} \quad x + 2 = 0 \\ x = 5 \quad \quad \quad x = -2 \end{aligned}$$



Interval	$x \leq -2$	$-2 \leq x \leq 5$	$5 \leq x$
Test Pt.	-5	0	10
Check	$-(-5)^2 + 3(-5) + 10$ $= -25 - 15 + 16$ $= -24$	$-(0)^2 + 3(0) + 10$ $= 0 - 0 + 10$ $= 10$	$-(10)^2 + 3(10) + 10$ $= -100 + 30 + 10$ $= -60$
So?	$-24 \leq 0$	$10 \geq 0$	$-60 \leq 0$

$\{x \mid x \leq -2, 5 \leq x, x \in \mathbb{R}\}$

Method 3 Case Analysis

$$\begin{aligned} -x^2 + 3x + 10 &\leq 0 \\ -(x - 5)(x + 2) &\leq 0 \\ (x - 5)(x + 2) &\geq 0 \end{aligned}$$

So either both factors have to be positive or both have to be negative.

$$\begin{aligned} x - 5 &\geq 0 & x + 2 &\geq 0 \\ x &\geq 5 & x &\geq -2 \end{aligned}$$

$$\begin{aligned} x - 5 &\leq 0 & x + 2 &\leq 0 \\ x &\leq 5 & x &\leq -2 \end{aligned}$$



$x \geq 5$

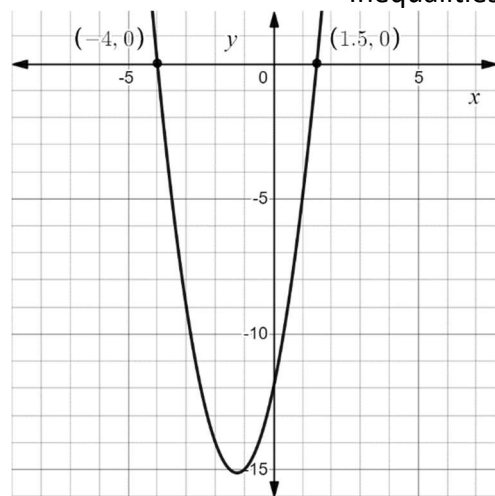
$x \leq -2$

$\{x \mid x \leq -2, 5 \leq x, x \in \mathbb{R}\}$

3. $2x^2 + 5x < 12$

Method 1 Graphing

$$\{x \mid -4 < x < 3/2, x \in \mathbb{R}\}$$



Method 2 Roots and Test Points

$$\begin{aligned} 2x^2 + 5x &= 12 \\ 2x^2 + 5x - 12 &= 0 \\ (2x - 3)(x + 4) &= 0 \\ 2x - 3 = 0 \quad \text{or} \quad x + 4 = 0 \\ x &= \frac{3}{2} \qquad \qquad \qquad x = -4 \end{aligned}$$



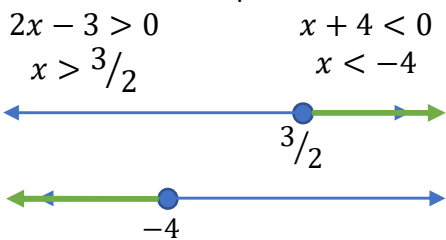
Interval	$x < -4$	$-4 < x < 3/2$	$3/2 < x$
Test Pt.	-5	0	5
Check	$2(-5)^2 + 5(-5)$ $= 50 - 25$ $= 25$	$2(0)^2 + 5(0)$ $= 0 - 0$ $= 0$	$2(5)^2 + 5(5)$ $= 50 + 25$ $= 75$
So?	$25 > 12$	$0 < 12$	$75 > 12$

$$\{x \mid -4 < x < 3/2, x \in \mathbb{R}\}$$

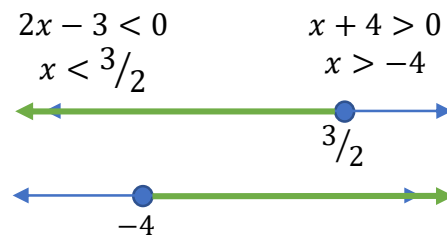
Method 3 Case Analysis

$$\begin{aligned} 2x^2 + 5x &< 12 \\ 2x^2 + 5x - 12 &< 0 \\ (2x - 3)(x + 4) &< 0 \end{aligned}$$

So one factor must be positive and the other must be negative.



No solutions



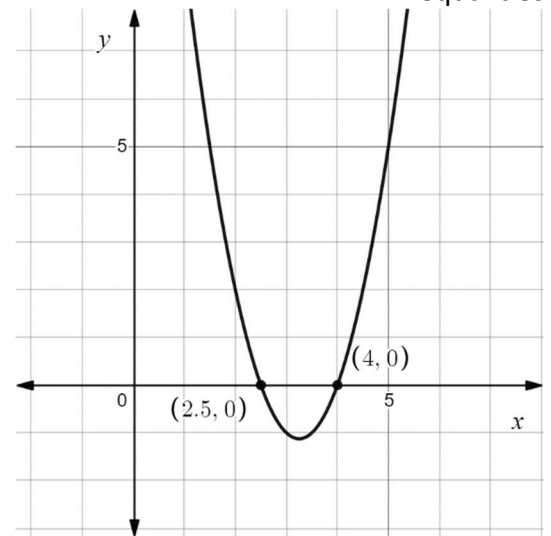
$$-4 < x < 3/2$$

$$\{x \mid -4 < x < 3/2, x \in \mathbb{R}\}$$

4. $x^2 - 5x \leq 3x^2 - 18x + 20$

Method 1 Graphing

$$\{x \mid \frac{5}{2} \leq x, 4 \leq x, x \in \mathbb{R}\}$$



Method 2 Roots and Test Points

$$\begin{aligned} x^2 - 5x &= 3x^2 - 18x + 20 \\ 0 &= 2x^2 - 13x + 20 \\ (2x - 5)(x - 4) &= 0 \\ 2x - 5 = 0 &\quad \text{or} \quad x - 4 = 0 \\ x = \frac{5}{2} &\quad \quad \quad x = 4 \end{aligned}$$



Interval	$x \leq \frac{5}{2}$	$\frac{5}{2} \leq x \leq 4$	$4 \leq x$
Test Pt.	0	3	5
Check	$(0)^2 - 5(0)$ $= 0 - 0$ $= 0$	$(3)^2 - 5(3)$ $= 9 - 15$ $= -6$	$(5)^2 - 5(5)$ $= 25 - 25$ $= 0$
	$3(0)^2 - 18(0) + 20$ $= 0 - 0 + 20$ $= 20$	$3(3)^2 - 18(3) + 20$ $= 27 - 54 + 20$ $= -7$	$3(5)^2 - 18(5) + 20$ $= 75 - 90 + 20$ $= 5$
So?	$0 \leq 20$	$-6 \geq -7$	$0 \leq 5$

$$\{x \mid \frac{5}{2} \leq x, 4 \leq x, x \in \mathbb{R}\}$$

Method 3 Case Analysis

$$\begin{aligned}
 x^2 - 5x &\leq 3x^2 - 18x + 20 \\
 0 &\leq 2x^2 - 13x + 20 \\
 0 &\leq (2x - 5)(x - 4)
 \end{aligned}$$

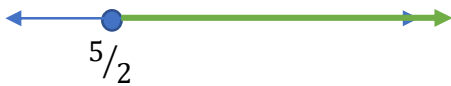
So either both factors have to be positive or both have to be negative.

$$\begin{aligned}
 2x - 5 &\geq 0 \\
 x &\geq 5/2
 \end{aligned}$$

$$\begin{aligned}
 x - 4 &\geq 0 \\
 x &\geq 4
 \end{aligned}$$

$$\begin{aligned}
 2x - 5 &\leq 0 \\
 x &\leq 5/2
 \end{aligned}$$

$$\begin{aligned}
 x - 4 &\leq 0 \\
 x &\leq 4
 \end{aligned}$$



$$x \geq 4$$

$$x \leq 5/2$$

$$\{x \mid 5/2 \leq x, 4 \leq x, x \in \mathbb{R}\}$$