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Chapter 1 Review
Functions and Models

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 1.1	Understanding new ideas about functions and applying that to previously knowledge.

1. Find the domain of the following functions.

Developing		
a. $f(x)=\dfrac{3x-5}{3x^2-8x+4}$ $\{x \mid x \neq {}^2/_3, 2, x \in \mathbb{R}\}$	b. $f(x)=\dfrac{2x+1}{4x^3+\ 11x^2+6x}$ $\{x \mid x \neq 0, -{}^3/_4, -2, x \in \mathbb{R}\}$	c. $f(x)=\dfrac{5x}{4x^2-17x+4}$ $\{x \mid x \neq {}^1/_4, 4, x \in \mathbb{R}\}$
d. $f(x)=\dfrac{x-5}{x^2+11x-80}$ $\{x \mid x \neq -16, 5, x \in \mathbb{R}\}$	e. $f(x)=\dfrac{7-x}{5x^2-40x+35}$ $\{x \mid x \neq 1, 7, x \in \mathbb{R}\}$	f. $f(x)=\dfrac{2}{9-3x-2x^2}$ $\{x \mid x \neq {}^3/_2, -3, x \in \mathbb{R}\}$
Proficient		
g. $f(x)=\sqrt{6x^2+5x-6}$ $\{x \mid x \leq -{}^3/_2, x \geq {}^2/_3, x \in \mathbb{R}\}$	h. $f(x)=\sqrt{5+15x-8x^2}$ $\{x \mid -{}^1/_4 \leq x \leq {}^5/_2, x \in \mathbb{R}\}$	i. $f(x)=\sqrt{3x^2-2x-5}$ $\{x \mid x \leq -1, x \geq {}^5/_3, x \in \mathbb{R}\}$
j. $f(x)=\dfrac{x-5}{\sqrt{6x^2+5x-6}}$ $\{x \mid x < -{}^3/_2, x > {}^2/_3, x \in \mathbb{R}\}$	k. $f(x)=\dfrac{7-x}{\sqrt{5+15x-8x^2}}$ $\{x \mid -{}^1/_4 < x < {}^5/_2, x \in \mathbb{R}\}$	l. $f(x)=\dfrac{2}{\sqrt{3x^2-2x-5}}$ $\{x \mid x < -1, x > {}^5/_3, x \in \mathbb{R}\}$

Extending	
m. $f(x)=\sqrt{x^2-5x+6}+\sqrt{3-x}$ $\{x \mid x \leq 1, x \geq 5, x \in \mathbb{R}\}$	n. $f(x)=\sqrt{x+2}-\sqrt[4]{18-x^2+3x}$ $\{x \mid -2 \leq x \leq 6, x \in \mathbb{R}\}$
o. $f(x)=\sqrt[3]{2x^2-3x-5}+\sqrt{-x+4}$ $\{x \mid x \leq 4, x \in \mathbb{R}\}$	p. $f(x)=\dfrac{\sqrt{x-2}}{x^2-9}$ $\{x \mid x \geq 2, x \neq 3, x \in \mathbb{R}\}$
q. $f(x)=\dfrac{\sqrt{1-2x-3x^2}}{x^2-1}$ $\{x \mid -{}^1/_3 \leq x < 1, x \in \mathbb{R}\}$	r. $f(x)=\dfrac{\sqrt[3]{x-2}}{x^2-9}$ $\{x \mid x \neq \pm 3, x \in \mathbb{R}\}$

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2. Find the function that results from the following function operations. State the domain.

$f(x) = x^2 - 3$ $\qquad\qquad$ $g(x) = \sqrt{2x}$

Developing		
a. $h(x) = (f * g)(x)$ $h(x) = x^2\sqrt{2x} - 3\sqrt{2x}$ $\{x \mid x \geq 0, x \in \mathbb{R}\}$	b. $h(x) = \left(\frac{f}{g}\right)(x)$ $h(x) = \frac{\sqrt{2x}(x^2 - 3)}{2x}$ $\{x \mid x > 0, x \in \mathbb{R}\}$	c. $h(x) = \left(\frac{g}{f}\right)(x)$ $h(x) = \frac{\sqrt{2x}}{x^2 - 3}$ $\{x \mid x \geq 0, x \neq \sqrt{3}, x \in \mathbb{R}\}$
d. $h(x) = (f + g)(x)$ $h(x) = x^2 + \sqrt{2x} - 3$ $\{x \mid x \geq 0, x \in \mathbb{R}\}$	e. $h(x) = (g \times f)(x)$ $h(x) = x^2\sqrt{2x} - 3\sqrt{2x}$ $\{x \mid x \geq 0, x \in \mathbb{R}\}$	f. $h(x) = (f - g)(x)$ $h(x) = x^2 - \sqrt{2x} - 3$ $\{x \mid x \geq 0, x \in \mathbb{R}\}$
Proficient		
g. $h(x) = (f \circ g)(x)$ $h(x) = 2x - 3$ $\{x \mid x \geq 0, x \in \mathbb{R}\}$	h. $h(x) = (g \circ f)(x)$ $h(x) = \sqrt{2(x^2 - 3)}$ $\{x \mid x \leq -\sqrt{3}, x \geq \sqrt{3}, x \in \mathbb{R}\}$	i. $h(x) = g(g(x))$ $h(x) = \sqrt[4]{2x}$ $\{x \mid x \geq 0, x \in \mathbb{R}\}$
j. $h(x) = f(f(x))$ $h(x) = x^4 - 6x^2 + 6$ $\{x \mid x \in \mathbb{R}\}$	k. $h(x) = g\left(f(f(x))\right)$ $h(x) = \sqrt{2(x^4 - 6x^2 + 6)}$ $\left\{x \,\middle\vert\, \begin{array}{c} x \leq -\sqrt{3 - \sqrt{3}}, \\ \sqrt{3 - \sqrt{3}} \leq x \leq -\sqrt{3 + \sqrt{3}}, \\ x \geq \sqrt{3 + \sqrt{3}}, \\ x \in \mathbb{R} \end{array}\right\}$	l. $h(x) = f\left(g(f(x))\right)$ $h(x) = x^2 - 9$ $\{x \mid x \leq -\sqrt{3}, x \geq \sqrt{3}, x \in \mathbb{R}\}$

3. Suppose that $f(x) = 3x - 4$. Find a function g such that the following are true.

Extending	
a. $(g \circ f)(x) = 5x + 2$ $g(x) = \frac{5}{3}x - \frac{26}{3}$	b. $(f \circ g)(x) = (3x + 1)(x + 5)$ $g(x) = x^2 + \frac{16}{3}x + 3$

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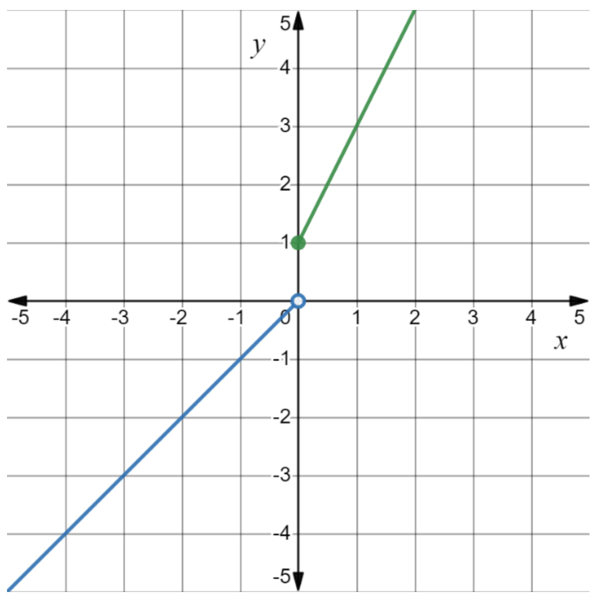
4. For each of the following functions, sketch the graph and find the domain and range.

Developing

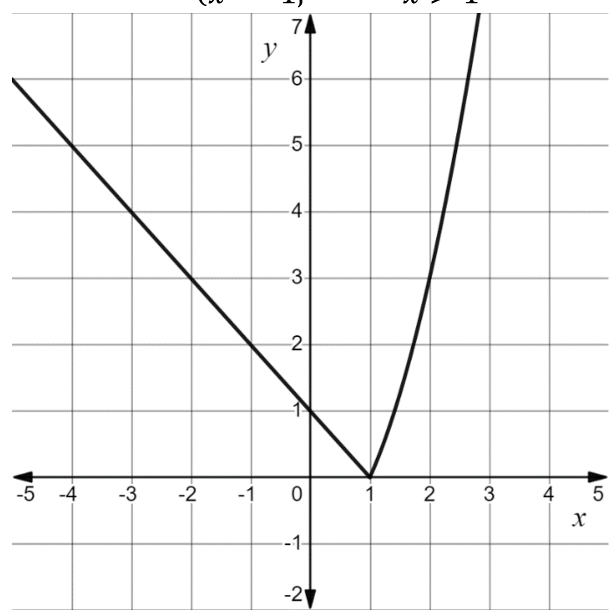
a. $f(x) = \begin{cases} x, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$



$\{x |\ x \epsilon \mathbb{R}\}$

$\{y | y < 0, y \geq 1, y \epsilon \mathbb{R}\}$

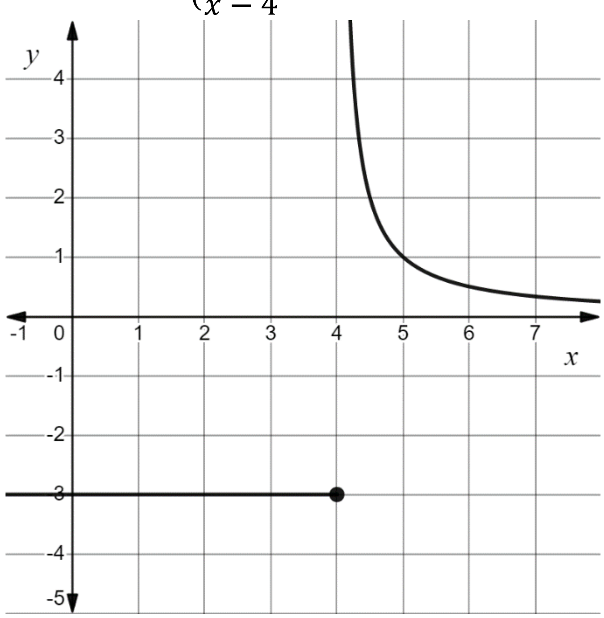
b. $f(x) = \begin{cases} -x + 1, & x \leq 1 \\ x^2 - 1, & x > 1 \end{cases}$



$\{x |\ x \epsilon \mathbb{R}\}$

$\{y | y \geq 0, y \epsilon \mathbb{R}\}$

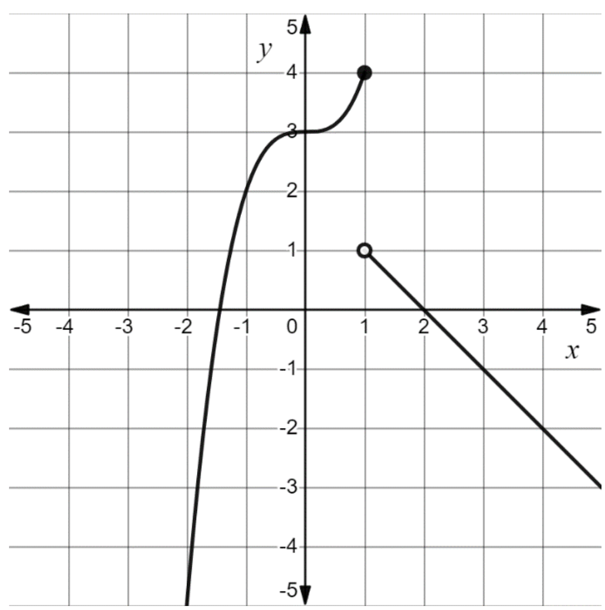
c. $f(x) = \begin{cases} -3, & x \leq 4 \\ \dfrac{1}{x - 4}, & x > 4 \end{cases}$



$\{x |\ x \epsilon \mathbb{R}\}$

$\{y | y = -3, y > 0, y \epsilon \mathbb{R}\}$

d. $f(x) = \begin{cases} x^3 + 3, & x \leq 1 \\ 2 - x, & x > 1 \end{cases}$



$\{x |\ x \epsilon \mathbb{R}\}$

$\{y | y \leq 4, y \epsilon \mathbb{R}\}$

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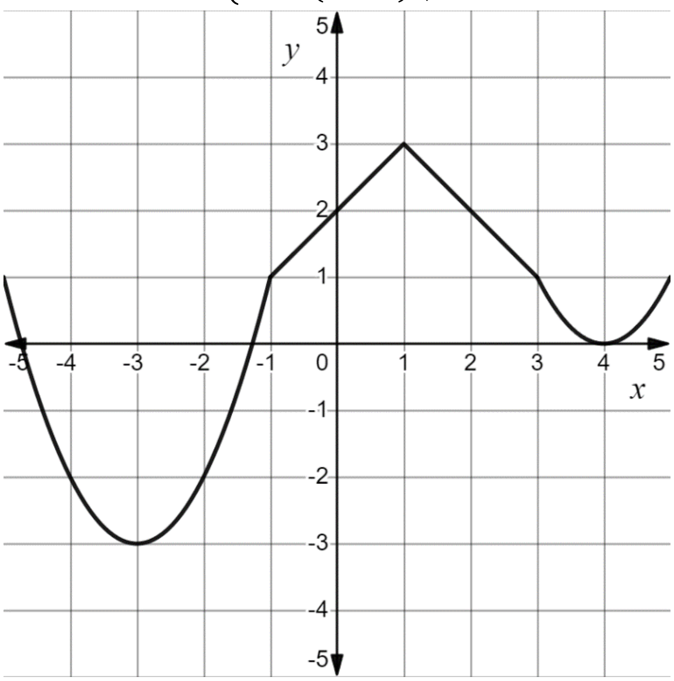
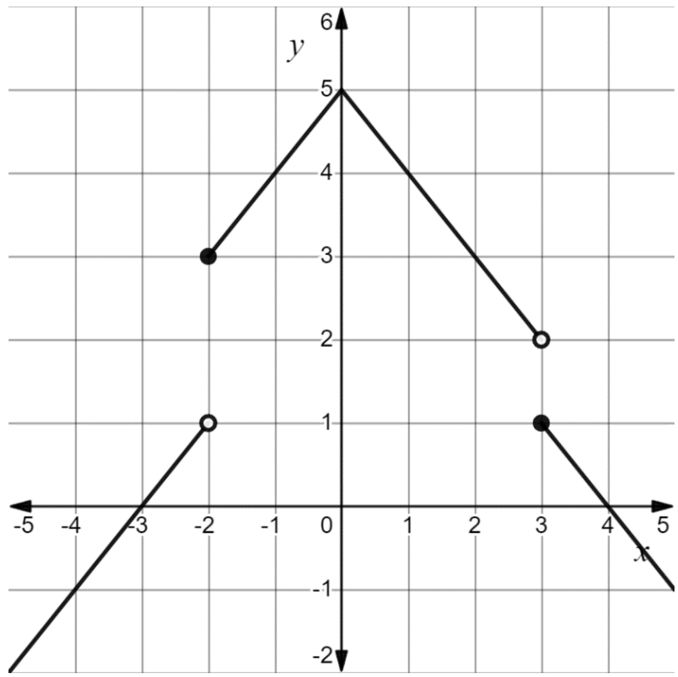
Proficient

e. $g(x) = \begin{cases} (x-3)^2 - 3, & x < -1 \\ -|x-1| + 3, & -1 \le x < 3 \\ (x-4)^2, & x \ge 3 \end{cases}$



$\{x |\ x \epsilon \mathbb{R}\}$
$\{y |\ y \epsilon \mathbb{R}\}$

f. $g(x) = \begin{cases} x + 3, & x < -2 \\ -|x| + 5, & -2 \le x < 3 \\ 4 - x, & x \ge 3 \end{cases}$



$\{x |\ x \epsilon \mathbb{R}\}$
$\{y | y \le 5, y \epsilon \mathbb{R}\}$

Extending

Please note all the functions you are responsible for, as well as any combination there of. This is not a full representation of what you are responsible for!

$y = x$ $\quad y = x^2$ $\quad y = x^3$

$y = \sqrt{x}$ $\quad y = |x|$ $\quad y = \frac{1}{x}$

$y = \sin x$ $\quad y = \cos x$ $\quad y = \tan x$

$y = 2^x$ $\quad y = \log_2 x$

Also, know the criteria for even and odd functions. Be confident in your algebra with respect to the difference quotient (this will be given to you).

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Learning Goal 1.2	Creating confidence in word problems.

Exponential/Logarithmic Models

1. A pump reduces the air pressure in a tank by 17% each second. Determine the function to model this situation and use it to determine the percent air pressure in the tank after 5 seconds. When will the air pressure be 50% of the starting pressure?

$$P(t) = 0.83^t$$
$$P(5) = 39\%$$
$$P(3.72) \approx 50\%$$

2. A computer, originally purchased for $\$3\ 000$ loses half its value every three years. Determine the model for the value of the computer, V as a function of time, t, and use this to find out approximately how long it will take for the computer to be worth 10% of its purchase price.

$$V(t) = 3000\left(\frac{1}{2}\right)^{t/3}$$
$$V(10) \approx 300$$

3. Lucas is hoping to take a vacation after he finishes university. To do this, he estimates he needs $\$5\ 000$. Lucas is able to finish his last year of university with $\$3\ 500$ in an investment that pays 8.4% compounded quarterly. How long will Lucas have to wait before he has enough money to take the vacation he wants?

$$4 \text{ years}, 3.5 \text{ months}$$

4. **A bacterial culture starts with $2\ 000$ bacteria and doubles every 0.75 hours. Determine the model of this situation and use it to find after how many hours will the bacteria count be $32\ 000$?**

$$N(t) = 2\ 000(2)^{4t/3}$$
$$N\left(\frac{15}{4}\right) = 32\ 000$$

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Trigonometric Models

5. Suppose a mass suspended on a spring is bouncing up and down. The mass's distance from the floor when it is at rest is 1 metre. The maximum displacement is 10 cm as it bounces. It takes 2 seconds to complete one bounce or cycle. Suppose the mass is at rest at $t = 0$ and that the spring bounces up first. Create a model that represents the displacement of the spring over time.

$$D(t) = 0.1\sin(\pi t) + 1$$

6. The number of hours of daylight, L, in Lethbridge, Alberta, may be modelled by a sinusoidal function of time, t. The longest day of the year is June 21, with 15.7 hours of daylight, and the shortest day is December 21, with 8.3 hours of daylight. Determine the model of this situation. How many hours of daylight are there on April 3?

$$S(t) = 3.7\cos\frac{2\pi}{365}(t - 172) + 12$$

$$S(93) = 12.8\text{ hours}$$

Geometric Models

7. A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 feet, express the area A of the window as a function of the width x of the window.

$$A = 15x - \left(\frac{\pi + 4}{8}\right)x^2$$

8. The diagonal of a rectangle is 17 cm long. The rectangle is 7 cm longer than it is wide. What is the area of the largest isosceles triangle inscribed inside the triangle?

$$60\text{ cm}^2$$