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Chapter 4a Review
Curve Sketching

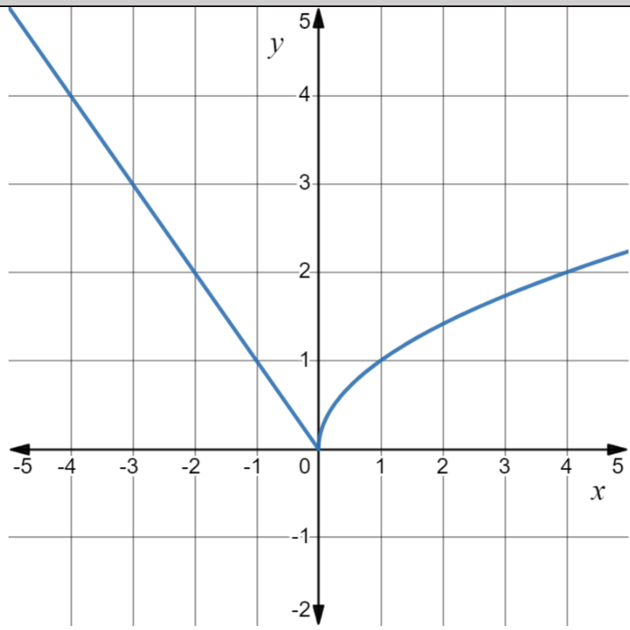
For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 4.1	Using derivative tests for curve sketching.

1. Given the graph of a function, determine the intervals where the function is increasing, constant and decreasing. Determine all extrema.

Developing

a.



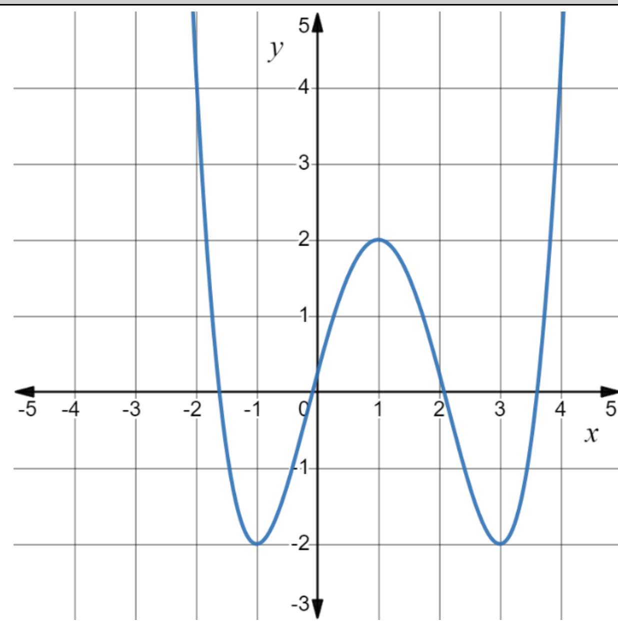
increasing $(0, \infty)$

decreasing $(-\infty, 0)$

constant –

extrema global min $(0, 0)$

b.



increasing $(-1, 1) \cup (3, \infty)$

decreasing $(-\infty, -1) \cup (1, 3)$

constant –

extrema global min $(-1, -2), (3, -2)$

local max $(1, 2)$

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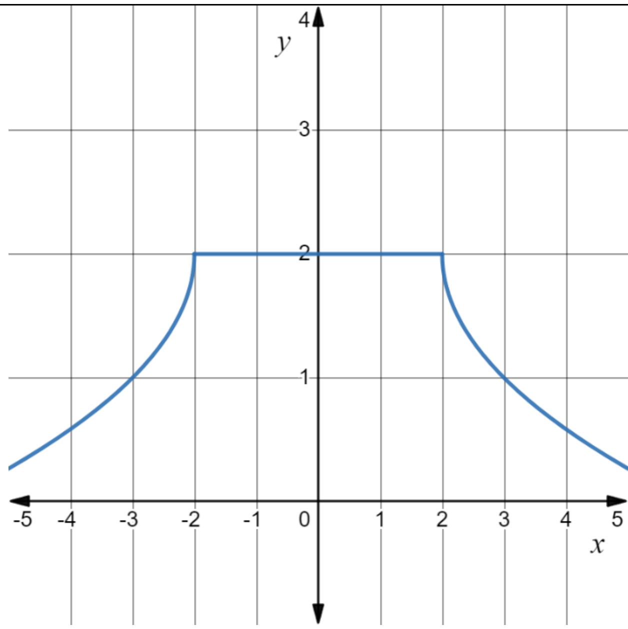
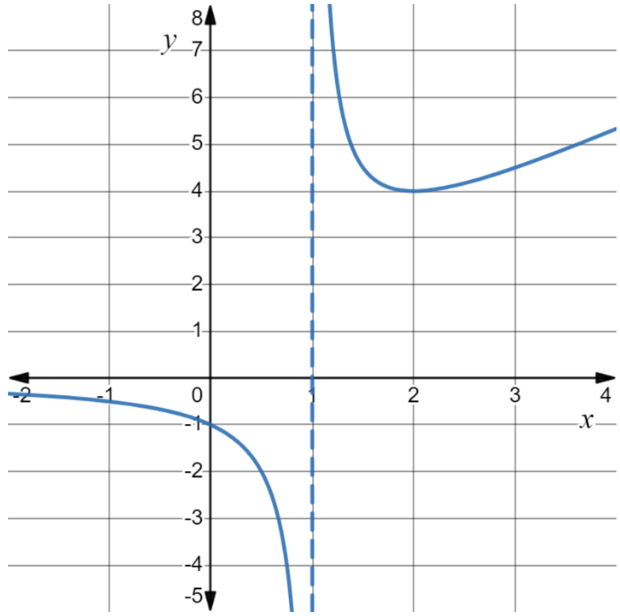
c.



increasing	$(-\infty, -2)$	
decreasing	$(2, \infty)$	
constant	$(-2, 2)$	
extrema	global max	$\{x \mid -2 < x < 2, x \in \mathbb{R}\}$, $y = 2$

d.



increasing	$(2, \infty)$	
decreasing	$(-\infty, 1) \cup (1, 2)$	
constant	$-$	
extrema	local min	$(2, 4)$

2. Given the following functions,
   i. Find all critical numbers,
   ii. Find the intervals for which the function is increasing/decreasing,
   iii. Find the intervals for which the function is concave up/down,
   iv. Find the inflection points, and
   v. Sketch the graph (remember to consider all asymptotes and holes).

Proficient

a. $f(x) = 4x^3 - 12x^2$

i. $x = 0, 2$

ii. inc $(-\infty, 0) \cup (0, 1)$
dec $(0, 2)$

iii. up $(1, \infty)$
down $(-\infty, 1)$

iv. $x = 1$

b. $g(x) = x^3 - 3x^2 - 24x + 32$

i.

ii.

iii.

iv.

c. $f(x) = x^{2/3}$

i.

ii.

d. $g(x) = x + \dfrac{1}{x}$

i.

ii.

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iii. iv.	iii. iv.
e. $f(x) = x^4 - 2x^2$ i. ii. iii. iv.	f. $g(x) = 2 + 3x - x^3$ i. ii. iii. iv.

Extending	
a. $f(x) = \dfrac{x^3}{1 - x^3}$ i. ii. iii. iv.	b. $g(x) = \dfrac{x^2 - 1}{x}$ v. vi. vii. viii.
c. $f(x) = \cos 2x - x$ i. ii. iii. iv.	d. $g(x) = \dfrac{5 - x}{x + 2}$ v. vi. vii. viii.
e. $f(x) = \dfrac{x^3}{x + 1}$ i. ii. iii. iv.	f. $g(x) = \sin^2 x$ v. vi. vii. viii.

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Learning Goal 4.2	The Mean Value Theorem and L'Hospital's Rule.

1. Show that the equation $2x - 1 - \sin x = 0$ has exactly one root.
2. Show that the equation $x^4 + 4x + c = 0$ has at most two real roots.
3. Suppose that $3 \leq f'(x) \leq 5$ for all values of x. Show that $18 \leq f(8) - f(2) \leq 30$.
4. Find all number c that satisfy the conclusion of the Mean Value Theorem for $f(x) = x^3 + x - 1$ on $[0, 2]$.

$$x = \frac{2\sqrt{3}}{3}$$

5. Let $f(x) = x^2$. Find a value $c \in (-1, 2)$ so that $f'(c)$ equals the slope between the endpoints of $f(x)$ on $[-1, 2]$.

$$c = \frac{1}{2}$$

6. Verify that the following function satisfies the hypothesis of the Mean Value Theorem on the interval $[-2, 6]$ and then find all of the values, c, that satisfy the conclusion of the theorem.

$$f(x) = \frac{3x}{x + 7}$$

7. Verify that the following function satisfies the hypothesis of the Mean Value Theorem on the interval $[1, 4]$ and then find all of the values, c, that satisfy the conclusion of the theorem.

$$f(x) = \frac{x}{x + 2}$$

$$c = \sqrt{18} - 2$$

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8. Evaluate the limit. Keep in mind when L'Hospital's Rule applies.

Proficient	
a. $\lim_{x\to 0}\frac{\tan \pi x}{\ln(1+x)}$ $=\pi$	b. $\lim_{x\to 0}\frac{e^{4x}-4x-1}{x^2}$ $=8$
c. $\lim_{x\to\infty} x^3 e^{-x}$ $=0$	d. $\lim_{x\to 0^+} x^2\ln x$ $=0$
e. $\lim_{x\to\pi}\frac{x^2-\pi^2}{\sin x}$ $=-2\pi$	f. $\lim_{x\to\infty}\frac{2x^2-3x+7}{x^2+47x+1}$ $=2$
g. $\lim_{x\to 0}\frac{\sec x-1}{\sin x}$ $=0$	h. $\lim_{x\to 0^+}\frac{1/x^2}{\ln x}$ $=-\infty$
i. $\lim_{x\to 0}\frac{\cos x-1}{\sin x}$ $=0$	j. $\lim_{x\to\infty}\frac{e^x}{x^3}$ $=\infty$
k. $\lim_{x\to\infty}\frac{\ln x}{x}$ $=0$	l. $\lim_{x\to 0}\frac{\sqrt{9+x}-3}{x}$ $=\frac{1}{6}$
m. $\lim_{x\to 2}\frac{2-\sqrt{x+2}}{4-x^2}$ $=\frac{1}{16}$	n. $\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$ $=\frac{3}{2}$
o. $\lim_{x\to 0}\frac{x^2}{\sqrt{2x+1}-1}$ $=0$	p. $\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$ $=\frac{3}{2}$
Extending	
a. $\lim_{x\to 1^+} x^{1/x-1}$ $=e$	b. $\lim_{x\to 0^+} x\ln x$ $=0$
c. $\lim_{x\to 0^+}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$ $=0$	d. $\lim_{x\to 0^+} x\ln x$ $=0$