

Name: _____

Date: _____

Learning Goal 5.2

Use exponent laws to evaluate expression with positive and negative rational exponents.

Recall:

$$1. 3^6 3^2 = 3^{6+2} \\ = 3^8$$

$$2. 6^3 6^7 6^2 6^5 = \\ = 6^{3+7+2+5} \\ = 6^{17}$$

$$3. 7^6 7^3 7^1 = \\ = 7^{6+3+1} \\ = 7^{10}$$

Extend the idea to non-whole number exponents:

$$4. 2^{1/2} 2^{1/2} = \\ = 2^{\frac{1}{2} + \frac{1}{2}} \\ = 2^1 \\ = 2$$

$$= (2^{\frac{1}{2}})^2 \\ = (2^2)^{\frac{1}{2}} \\ = 2$$

$$5. 5^{0.25} 5^{0.25} 5^{0.25} 5^{0.25} = \\ = 5^{0.25+0.25+0.25+0.25} \\ = 5^1 \\ = 5$$

$$(5^{0.25})^4 \\ = (5^4)^{0.25}$$

$$6. 11^{1/3} 11^{1/3} 11^{1/3} = \\ = 11^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} \\ = 11^1 \\ = 11$$

Take a silent moment. What do you think the **fractional exponents** represent?RADICALS !!
j

If you have a fractional exponent

- the numerator is the regular exponent
- the denominator is the index of the radical.

$$-8 \neq \frac{1}{8}$$

Example Write each power as a radical, then simplify.

$$\begin{aligned} 1. \quad 1000^{1/3} \\ &= \sqrt[3]{1000} \\ &= 10 \end{aligned}$$

$$\begin{aligned} 2. \quad 0.25^{-1/2} \\ &= \left(\frac{1}{4}\right)^{-1/2} \\ &= (4)^{1/2} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} 3. \quad (-8)^{1/3} \\ &= \sqrt[3]{-8} \\ &= \sqrt[3]{-2 \times -2 \times -2} \\ &= -2 \end{aligned}$$

$$\begin{aligned} 4. \quad \left(\frac{16}{81}\right)^{-1/4} \\ &= \left(\frac{81}{16}\right)^{1/4} \\ &= \sqrt[4]{\frac{81}{16}} \\ &= \frac{3}{2} \end{aligned}$$

What if the exponent is not a unit fraction? Take a silent minute to consider.

$$40^{2/3} \qquad 40 < \begin{matrix} 4 < 2 \\ 10 < 5 \end{matrix}$$

$\begin{aligned} (40^2)^{1/3} &= \sqrt[3]{40^2} \\ &= \sqrt[3]{1600} \\ &= \sqrt[3]{2^4 \cdot 5^2} = 2^2 \sqrt[3]{5^2} \end{aligned}$	$\begin{aligned} (40^{1/3})^2 &= (\sqrt[3]{40})^2 \\ &= (\sqrt[3]{2^3 \cdot 5})^2 \\ &= (2\sqrt[3]{5})^2 = 2^2 \sqrt[3]{5^2} \end{aligned}$
---	---

* the numerator is just a regular exponent *

Example Write each power as a radical, then simplify.

$$\begin{aligned} 1. \quad 0.01^{3/2} \\ &= \left(\frac{1}{100}\right)^{3/2} \\ &= \left(\sqrt{\frac{1}{100}}\right)^3 \\ &= \left(\frac{1}{10}\right)^3 \\ &= \frac{1}{1000} \end{aligned}$$

$$\begin{aligned} 2. \quad (-27)^{4/3} \\ &= (\sqrt[3]{-27})^4 \\ &= (-3)^4 \\ &= 3^4 \\ &= 81 \end{aligned}$$

$27 < \frac{9}{3} = \frac{3}{3} = 3^3$

$$\begin{aligned} 3. \quad 81^{-3/4} \\ &= \left(\frac{1}{81}\right)^{3/4} \\ &= \left(\sqrt[4]{\frac{1}{81}}\right)^3 \\ &= \left(\frac{1}{3}\right)^3 \\ &= \frac{1^3}{3^3} = \frac{1}{27} \end{aligned}$$

$1.2 = \frac{6}{5}$

$$\begin{aligned} 4. \quad 0.75^{-1.2} \\ &= \left(\frac{3}{4}\right)^{-6/5} \\ &= \left(\frac{4}{3}\right)^{6/5} > 1 \\ &= \sqrt[5]{\left(\frac{4}{3}\right)^6} \\ &= \sqrt[5]{\frac{4^6}{3^6}} = \frac{4}{3} \sqrt[5]{\frac{4}{3}} \end{aligned}$$