Name: ___________________________ Date: ____________________

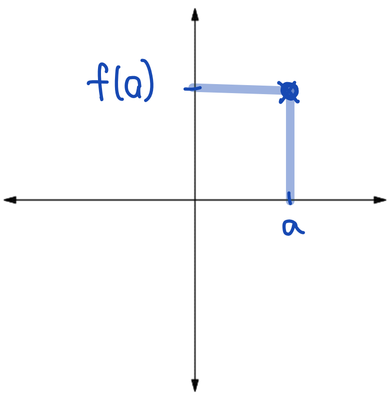
Learning Goal 2.1	Finite limits and continuity.

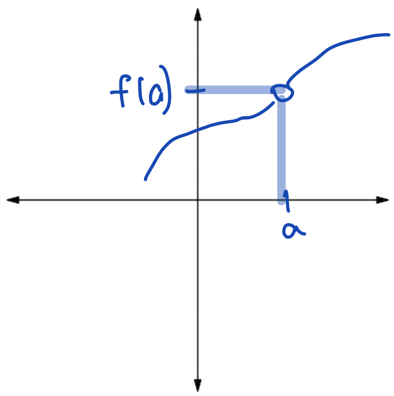
Continuity

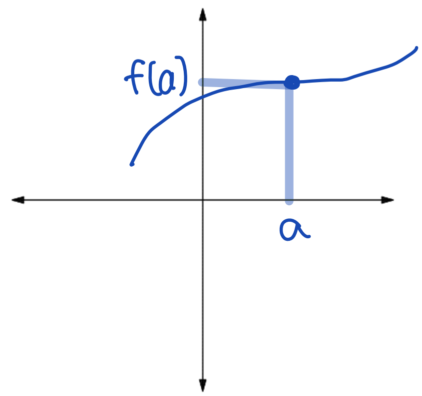
- no holes
- no asymptotes
- no broken parts

$\lim_{x \to a} f(x) = f(a)$

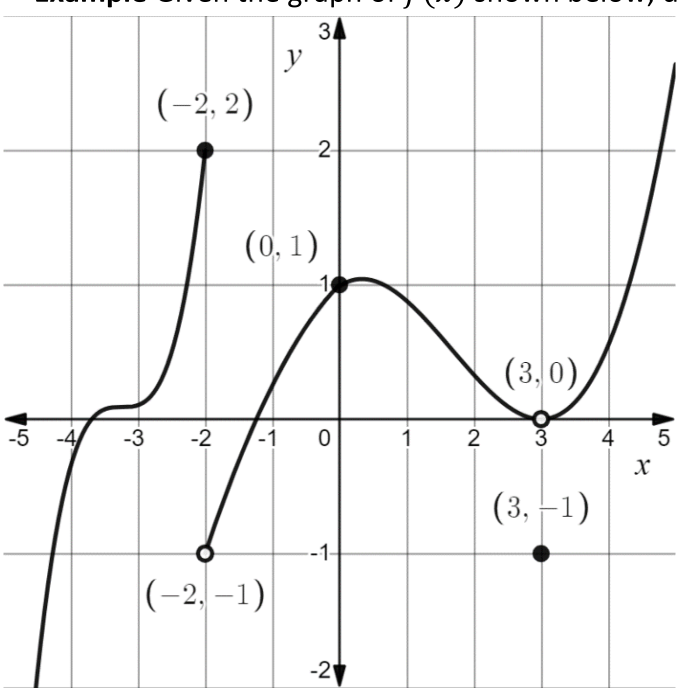
1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ must exist
3. $\lim_{x \to a} f(x) = f(a)$







Example Given the graph of $f(x)$ shown below, determine if $f(x)$ is continuous at $x = -2, 0$ and 3.



$x = -2$ $\quad f(-2) = 2$

$$\lim_{x \to -2} f(x) = \begin{cases} \lim_{x \to -2^-} f(x) = 2 \\ \lim_{x \to -2^+} f(x) = -1 \end{cases}$$

$= DNE$

$\Rightarrow$ not continuous @ $x = -2$

$x = 0$ $\quad \left.\begin{array}{l} f(0) = 1 \\ \lim_{x \to 0} f(0) = 1 \end{array}\right\}$ $\Rightarrow$ continuous @ $x = 0$

$x = 3$ $\quad \left.\begin{array}{l} f(3) = -1 \\ \lim_{x \to 3} f(x) = 0 \end{array}\right\}$ $\Rightarrow$ not continuous at $x = 3$

Example Determine where the functions are not continuous, if anywhere.

a. $f(x) = \dfrac{x^2 - x - 2}{x - 2}$

$= \dfrac{\cancel{(x-2)}(x+1)}{\cancel{x-2}}$ ↖ hole at $x = 2$

$= x+1$ $\Rightarrow$ not continuous at $x=2$

b. $g(x) = \dfrac{4x + 10}{x^2 - 2x - 15}$

$= \dfrac{2(2x+5)}{(x-5)(x+3)}$ $\Rightarrow$ discontinuous at $x = -3, 5$

VA @ $x = -3, 5$

c. $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$ $\quad x = 2$

$\lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+1)}{\cancel{x-2}} = \lim\limits_{x\to 2} x+1$

$= 2+1$

$= 3$

$f(2) = 3$

$\lim\limits_{x\to 2} f(x) = f(2) \Rightarrow$ continuous.

d. $h(x) = \begin{cases} x + 1, & x \leq 1 \\ \dfrac{1}{x}, & 1 < x < 3 \\ \sqrt{x - 3}, & x \geq 3 \end{cases}$ $\quad x = 1, 3$

$f(1) = 1+1 = 2$

$\lim\limits_{x\to 1^-} x+1 = 2 \qquad \lim\limits_{x\to 1^+} \dfrac{1}{x} = 1$

$\Rightarrow$ not continuous (limit doesn't exist)

$f(3) = \sqrt{3-3} = 0$

$\lim\limits_{x\to 3^-} \dfrac{1}{x} = \dfrac{1}{3} \qquad \lim\limits_{x\to 3^+} \sqrt{x-3} = 0$

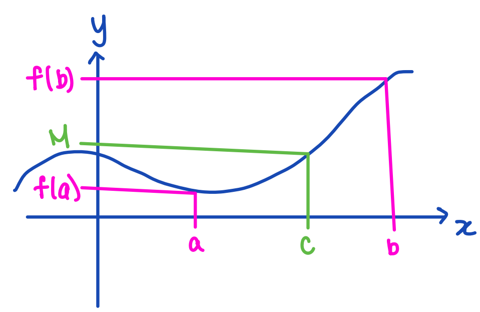
$\Rightarrow$ not continuous (limit doesn't exist)

Types of Discontinuity

1. Essential or infinite. $\lim\limits_{x\to a} f(x) = \pm\infty$ * vertical asymptote
2. Jump $\lim\limits_{x\to a^-} f(x) \neq \lim\limits_{x\to a^+} f(x)$ example d
3. Point example c-ish
4. Removable example a

Intermediate Value Theorem ↙ $\{x \mid a \leq x \leq b, x \in \mathbb{R}\}$

Let $f(x)$ be continuous on $[a,b]$ and let M be any value between $f(a)$ and $f(b)$. Then there must exist a 'c' such that $a < c < b$ and $f(c) = M$.



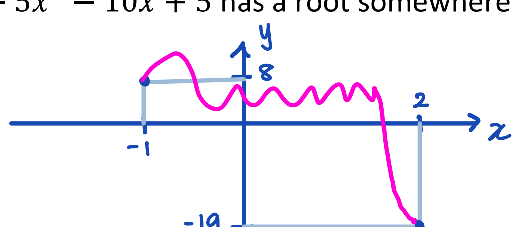
↙ zero, x-intercept.

Example Show that $p(x) = 2x^3 - 5x^2 - 10x + 5$ has a root somewhere in the interval $[-1, 2]$.

$P(-1) = 2(-1)^3 - 5(-1)^2 - 10(-1) + 5$

$= -2 - 5 + 10 + 5$

$= 8$



$P(2) = 2(2)^3 - 5(2)^2 - 10(2) + 5$

$= 16 - 20 - 20 + 5$

$= -19$