Name: _________________________ Date: ___________________

Learning Goal 2.1	Finite limits and continuity.

More Questions – Solutions

1. What value of c will make the following function $f(x)$ continuous at 2?

$$f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2}, & x \neq 2 \\ c, & x = 2 \end{cases}$$

$$\begin{aligned} \frac{x^2 - x - 2}{x - 2} &= \frac{(x-2)(x+1)}{x-2} \\ &= x + 1 \\ &= 3 \text{ when } x = 2 \\ c &= 3 \end{aligned}$$

Theorem of Continuity of Function Composition
If g is continuous at a and f is continuous at $g(a)$ then the composition $f \circ g$ is continuous at a.

2. Determine where the following function are continuous.

a. $h(x) = \cos(x^2)$

$$h(x) = f(g(x))$$

$g(x) = x^2$ is continuous everywhere (like all polynomials)

$f(x) = \cos x$ is also continuous everywhere (like all sinusoidal curves)

So $h(x)$ is continuous over all real numbers.

b. $h(x) = \ln(1 + \sin x)$

$$h(x) = f(g(x))$$

$g(x) = 1 + \sin x$ is also continuous everywhere (like all sinusoidal curves)

$f(x) = \ln x$ Is only defined when $x > 0$

We can only use values where $g(x) > 0$.

$$0 \leq 1 + \sin x \leq 2$$

So we need to find the minimums of $g(x)$ and exclude those from the domain.

$$\left\{ x \,\middle|\, x \neq \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}, x \in \mathbb{R} \right\}$$

3. Show there are solutions to the following equations in the given intervals.

a. $f(x) = \sqrt[3]{x} + x - 1$ $(0, 8)$

$$f(0) = -1$$
$$f(8) = 9$$

Because there is one positive and one negative value, the function must have crossed the $x-$ axis at least once in this interval by the intermediate value theorem.

b. $g(x) = x^3 + 3x^2 + x - 2$ $(0, 1)$

$$g(0) = -2$$
$$g(1) = 3$$

Because there is one positive and one negative value, the function must have crossed the $x-$ axis at least once in this interval by the intermediate value theorem.

Assignment # 2 – 5, 13 – 23, 39 – 43, 49 – 55 Quiz Next Day!