Name: ________________________ Date: __________________

Learning Goal 3.7	Creating confidence in word problems.

Exponential Growth and Decay

Let $y = f(t)$ represents a population size at time t.

THERE'S ONLY ONE POSSIBLE FUNCTION FOR THIS. $f(t) = e^{kt}$

$$f'(t) = k \times f(t)$$

$k > 0$

e^{kt} (raise a # to a +ve exp)

GROWTH

$k < 0$

$e^{-kt} = \frac{1}{e^{kt}}$

Decay

IC

Example Use the fact that the world population was 2 560 million in 1950 and 3 040 million in 1960 to model the population of the world in the second half of the 20th century (assuming the growth rate is proportional to the population size). What is the relative growth rate? Use the model to estimate the world population in 1993 and to predict the population in the year 2023.

$\boxed{\frac{dP}{dt} = kP}$

t=0
1950

$P(t) = P(0)e^{kt}$
$= 2560e^{kt}$

initial conditions

1960
t=10

$P(10) = \frac{2560e^{k(10)}}{2560} = \frac{\cancel{3040}}{\cancel{2560}}$ $\frac{19}{16}$

$\ln(e^{10k}) = \ln\left(\frac{19}{16}\right)$

$10k = \ln\left(\frac{19}{16}\right)$

$k = \frac{\ln\left(\frac{19}{16}\right)}{10}$

$\doteq 0.017$

$\ln x = \log_e x$

THE GROWTH RATE IS 1.7% PER YEAR.

1993
t=43

$P(43) = 2560e^{0.017(43)}$
$= 5.36$ Billion

2023
t=73

$P(73) = 2560e^{0.017(73)}$
$= 9.0$ Billion

But it's only 8 so our pop. rate has slowed!

Assignment # 1 – 15, 19 Quiz Next Day!

$A = A_0(r)^{k/t}$ k is a GROWTH Rate.

Example The half – life of radium−226 is 1 590 years.

a. A sample of radium−226has a mass of 100 mg. Find a formula for the mass of the sample that remains after t years.

$$m(t) = 100e^{kt}$$
$$m(1590) = 100e^{k(1590)} = 50$$
$$\ln(e^{1590k}) = \ln(0.5)$$
$$1590k = \ln(0.5)$$
$$k = \frac{\ln(0.5)}{1590} \doteq -0.00044$$

$$m(t) = 100e^{t\ln(0.5)/1590}$$

b. Find the mass after 1 000 years correct to the nearest milligram.

$$m(1000) = 100e^{1000\ln(0.5)/1590}$$
$$\doteq 65 \text{ mg.}$$

c. When will the mass be reduced to 30 mg?

$$m(t) = 100e^{t\ln(0.5)/1590} = 30$$
$$\ln(e^{t\ln(0.5)/1590}) = \ln(0.3)$$
$$\frac{t\ln(0.5)}{1590} = \ln(0.3)$$
$$t = \frac{1590\ln(0.3)}{\ln(0.5)}$$

Example If $1 000 is invested, how much will the investment be worth after 3 years at 6% interest if

a. It is compounded annually? $n=1$

$$A(3) = 1000\left(1+\frac{0.06}{1}\right)^{1\times3}$$
$$= 1000(1.06)^3$$
$$= \$1\,191.02$$

b. It is compounded monthly? $n=12$

$$A(3) = 1000\left(1+\frac{0.06}{12}\right)^{12\times3}$$
$$= 1000(1.005)^{36} = \$1\,196.68$$

c. It is compounded daily? $n=365$

$$A(3) = 1000\left(1+\frac{0.06}{365}\right)^{365\times3}$$
$$= 1000(1.00016)^{1095}$$
$$= \$1\,197.20$$

d. It is compounded continuously?

$$A(3) = 1000(e)^{0.06\times3}$$
$$= \$1\,197.22$$

initial investment

rate

time

$$A = A_0\left(1+\frac{r}{n}\right)^{nt}$$

COMPOUNDING PERIOD