

Name: _____

Date: _____

Learning Goal 3.7

Creating confidence in word problems.

More Questions – Solutions

1. A bottle of soda pop at room temperature (72°F) is placed in a refrigerator where the temperature is 44°F. After half an hour the soda pop has cooled to 61°F.

$$61 = 72e^{r(30)}$$

$$\frac{61}{72} = e^{30r}$$

$$\ln\left(\frac{61}{72}\right) = 30r$$

$$r = \frac{1}{30} \ln\left(\frac{61}{72}\right)$$

$$\begin{aligned} T_F &= 72e^{1/30 \ln(61/72)t} \\ &= 72\left(e^{\ln(61/72)}\right)^{1/30t} \\ &= 72\left(\frac{61}{72}\right)^{t/30} \end{aligned}$$

- a. What is the temperature of the soda pop after another half hour?

$$\begin{aligned} T_F &= 72\left(\frac{61}{72}\right)^{60/30} \\ &= 72\left(\frac{61}{72}\right)^2 \\ &= \frac{61^2}{72} \\ &\approx 52^\circ\text{F} \end{aligned}$$

- b. How long does it take for the soda pop to cool to 50°F?

$$\begin{aligned} 50 &= 72\left(\frac{61}{72}\right)^{t/30} \\ \frac{50}{72} &= \left(\frac{61}{72}\right)^{t/30} \\ \ln\left(\frac{50}{72}\right) &= \ln\left(\frac{61}{72}\right)^{t/30} \\ \ln\left(\frac{50}{72}\right) &= \frac{t}{30} \ln\left(\frac{61}{72}\right) \\ \frac{t}{30} &= \frac{\ln\left(\frac{50}{72}\right)}{\ln\left(\frac{61}{72}\right)} \\ t &= 30 \frac{\ln\left(\frac{50}{72}\right)}{\ln\left(\frac{61}{72}\right)} \approx 66 \text{ minutes} \end{aligned}$$

2. How long will it take an investment to double in value if the interest rate is 6% compounded continuously? What is the equivalent annual interest rate, compounded monthly?

$$\begin{aligned}
 A &= A_0 e^{rt} \\
 \frac{A}{A_0} &= e^{0.06t} \\
 2 &= e^{0.06t} \\
 \ln 2 &= \ln(e^{0.06t}) \\
 \ln 2 &= 0.06t \\
 t &= \frac{\ln 2}{0.06}
 \end{aligned}$$

$$\begin{aligned}
 A &= A_0 \left(1 + \frac{r}{n}\right)^{nt} \\
 2 &= \left(1 + \frac{r}{12}\right)^{12(\ln 2/0.06)} \\
 \ln(2) &= \ln\left(1 + \frac{r}{12}\right)^{12(\ln 2/0.06)} \\
 \ln(2) &= 12 \left(\frac{\ln 2}{0.06}\right) \ln\left(1 + \frac{r}{12}\right) \\
 1 &= 12 \left(\frac{1}{0.06}\right) \ln\left(1 + \frac{r}{12}\right) \\
 \frac{1}{200} &= \ln\left(1 + \frac{r}{12}\right) \\
 e^{1/200} &= 1 + \frac{r}{12} \\
 r &= 12(e^{1/200} - 1)
 \end{aligned}$$

3. A freshly brewed cup of coffee has temperature of 95°C in a 20°C room. When its temperature is 70°C, it is cooling at a rate of 1°C per minute. When does this occur?

$$\begin{aligned}
 70 &= 95 e^{-t} \\
 \frac{70}{95} &= e^{-t} \\
 \ln\left(\frac{70}{95}\right) &= -t \\
 \ln\left(\frac{95}{70}\right) &= t \\
 t &= \ln\left(\frac{19}{14}\right)
 \end{aligned}$$