Name: _____ Date: _____

Learning Goal 3.2	Applying derivatives to trigonometric and exponential functions.

More Questions – Solutions

1. Use the quotient rule to find the derivatives.

a. $y = \sec x$

$$y = \frac{1}{\cos x}$$

$$\frac{dy}{dx} = \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x}$$
$$= \frac{\sin x}{\cos^2 x}$$
$$= \tan x \sec x$$

b. $y = \csc x$

$$y = \frac{1}{\sin x}$$

$$\frac{dy}{dx} = \frac{(\sin x)(0) - (1)(\cos x)}{\sin^2 x}$$
$$= \frac{\cos x}{\sin^2 x}$$
$$= \cot x \csc x$$

c. $y = \cot x$

$$y = \frac{\cos x}{\sin x}$$

$$y' = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x}$$
$$= \frac{\cos^2 x - \sin^2 x}{\sin^2 x}$$
$$= \cot^2 x - 1$$

NOTE Now that we know what these are we can use them as a rule. We don`t need to derive them each time. On the other hand, be confident that you can if your memory fails you!

2. Find the following derivatives.

a. $g(x) = 3\sec x - 10\cot x$

$$g'(x) = 3\tan x \sec x - 10(\cot^2 x - 1)$$

b. $y = 5\sin x \cos x + 4\csc x$

$$\frac{dy}{dx} = 5(\cos^2 x - \sin^2 x) + 4\cot x \csc x$$

c. $c(w) = \frac{3}{w^4} - w^2 \tan w$

$$c'(w) = -12w^{-5} - (w^2 \sec^2 w + 2w \tan w)$$
$$= -\frac{12}{w^5} - w(w\sec^2 w + 2\tan w)$$

d. $h(x) = (x + \sin(x^2))^{10}$

$$h'(x) = 10(x + \sin(x^2))^9 \times (1 + 2x\cos(x^2))$$

e. $k(x) = \sin(\cos^2 x)$

$$k'(x) = \cos(\cos^2 x) \times 2\cos x \times -\sin x$$
$$= -2\sin x \cos x \cos(\cos^2 x)$$

f. $h(s) = \sin\sqrt{s^2 - 1}$

$$h'(s) = \cos\sqrt{s^2 - 1} \times \frac{1}{2\sqrt{s^2 - 1}} \times 2s$$
$$h'(s) = \frac{s \times \cos\sqrt{s^2 - 1}}{\sqrt{s^2 - 1}}$$

3. Find an equation of the tangent line to the graph of the function $f(x) = \tan 2x$ at the point $\left(\pi/8, 1\right)$.

$$\begin{aligned} f'(x) &= 2\sec^2 2x \\ f'\left(\frac{\pi}{8}\right) &= 2\sec^2 2\left(\frac{\pi}{8}\right) \\ &= 2\sec^2\left(\frac{\pi}{4}\right) \\ &= 2\left(\sqrt{2}\right)^2 \\ &= 4 \end{aligned}$$

$$y - 1 = 4\left(x - \frac{\pi}{8}\right)$$

4. Find the points on the curve $y = x + 2\cos x$ that have a horizontal tangent line.

$$\begin{aligned} \frac{dy}{dx} &= 1 - 2\sin x \\ 0 &= 1 - 2\sin x \\ -1 &= -2\sin x \\ \frac{1}{2} &= \sin x \\ x &= \frac{\pi}{6}, \frac{5\pi}{6} \end{aligned}$$

$$x = \frac{\pi}{6} + 2\pi n, n \in \mathbb{Z}$$
$$x = \frac{5\pi}{6} + 2\pi n, n \in \mathbb{Z}$$