Name: _____ Date: _____

Learning Goal 3.3	Solving equations algebraically and graphically.

More Questions - Solutions

1. Consider the equation $f(x) = x^3 + 4x^2 + 5x + 2$.

Degree	Leading Coefficient	Constant	Domain	$y-$ intercept	Number of $x-$ intercepts
Three	1	2	$\{x \mid x \in \mathbb{R}\}$	$y = 2$ $(0, 2)$	1, 2 or 3

a. What is the end behaviour of the function?

$$\lim_{x \to -\infty} f(x) = -\infty \qquad \lim_{x \to \infty} f(x) = \infty$$

b. Factor the equation.

There are very few options here: ± 1 and ± 2. So let's try $x - 2$ first.

$$f(2) = (2)^3 + 4(2)^2 + 5(2) + 2 = 8 + 16 + 10 + 2 = 36 \neq 0$$

$$f(-2) = (-2)^3 + 4(-2)^2 + 5(-2) + 2 = -8 + 16 - 10 + 2 = 0$$

-2	1	4	5	2
		-2	-4	-2
	1	2	1	0

$$\begin{aligned} f(x) &= x^3 + 4x^2 + 5x + 2 \\ &= (x+2)(x^2 + 2x + 1) \\ &= (x+2)(x+1)(x+1) \\ &= (x+2)(x+1)^2 \end{aligned}$$

c. Find the solutions or roots of the equation. What multiplicity does each have?

$$(x+2)(x+1)^2 = 0$$
$$x = -2, -1$$

$x = -2$ has multiplicity one, and $x = -1$ has multiplicity two.

2. Consider the equation $g(x) = 10x^3 + 19x^2 + 2x - 7$.

Degree	Leading Coefficient	Constant	Domain	$y-$ intercept	Number of $x-$ intercepts
Three	10	-7	$\{x \mid x \in \mathbb{R}\}$	$y = -7$ $(0, -7)$	1, 2 or 3

d. What is the end behaviour of the function?

$$\lim_{x \to -\infty} f(x) = -\infty \qquad \lim_{x \to \infty} f(x) = \infty$$

e. Factor the equation.

There are **many** options here, but let's focus on the constant term: ± 1 and ± 7.
So let's try $x - 1$ first.

$$\begin{aligned} f(1) &= 10(1)^3 + 19(1)^2 + 2(1) - 7 \\ &= 10 + 19 + 2 - 7 \\ &= 24 \\ &\neq 0 \end{aligned}$$

$$\begin{aligned} f(-1) &= 10(-1)^3 + 19(-1)^2 + 2(-1) - 7 \\ &= -10 + 19 - 2 - 7 \\ &= 0 \end{aligned}$$

$$\begin{array}{r|cccc} -1 & 10 & 19 & 2 & -7 \\ & & -10 & -9 & 7 \\ \hline & 10 & 9 & -7 & 0 \end{array}$$

$$\begin{aligned} f(x) &= 10x^3 + 19x^2 + 2x - 7 \\ &= (x+1)(10x^2 + 9x - 7) \end{aligned}$$

$$\begin{aligned} -5 \times 14 &= -70 \\ -5 + 14 &= 9 \end{aligned}$$

$$\begin{aligned} &= (x+1)(10x^2 - 5x + 14x - 7) \\ &= (x+1)\big(10x(2x-1) + 7(2x-1)\big) \\ &= (x+1)(2x-1)(10x+7) \end{aligned}$$

f. Find the solutions or roots of the equation. What multiplicity does each have?

$$(x+1)(2x-1)(10x+7) = 0$$
$$x = -1, \frac{1}{2}, -\frac{7}{10}$$

All the roots have multiplicity one.