Name: ___________________________ Date: ____________________

Learning Goal 3.3	Creating confidence in (baby) word problems.

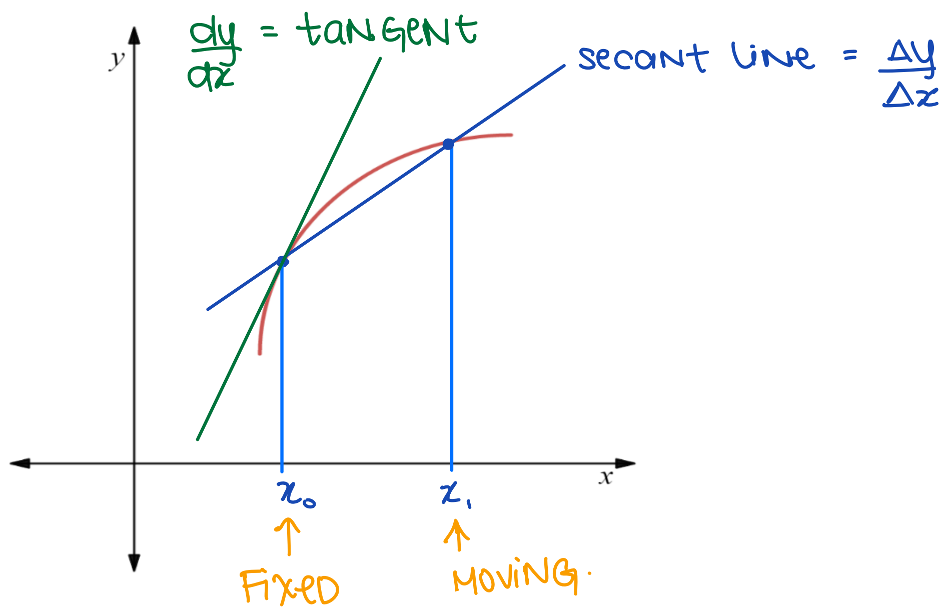
There are two kinds of rate of change that we use to solve application problems:

1. Average Rate of Change
   - secant line between 2 points.

   $$\frac{\Delta y}{\Delta x} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \quad \left(= \frac{y_2 - y_1}{x_2 - x_1} \right)$$

2. Instantaneous Rate of Change.
   - tangent line at the point of interest.

   $$\frac{dy}{dx} = \lim_{x_1 \to x_0} \frac{\Delta y}{\Delta x} = f'(x_0)$$



Most Common Physical Example

distance / displacement $= d(t)$

velocity $= v(t) = d'(t)$

acceleration $= a(t) = v'(t) = d''(t)$

jerk $= j(t) = a'(t) = v''(t) = d'''(t)$

Example The position of a particle is given by the equation $d(t) = t^3 - 6t^2 + 9t$

a. What is the velocity of the particle at any time t?

$v(t) = d'(t)$
$= 3t^2 - 12t + 9$

b. What is the velocity of the particle after 2 seconds? After 4 seconds?

$v(2) = 3(2)^2 - 12(2) + 9$
$= 12 - 24 + 9 = -3$ units/sec

$v(4) = 3(4)^2 - 12(4) + 9$
$= 48 - 48 + 9 = 9$ units/sec

Use the distance function!

c. What is the average velocity of the particle from 2 seconds to 4 seconds?

$$\frac{\Delta d}{\Delta t} = \frac{d(4) - d(2)}{4 - 2}$$
$$= \frac{\left((4)^3 - 6(4)^2 + 9(4)\right) - \left((2)^3 - 6(2)^2 + 9(2)\right)}{2}$$
$$= \frac{4 - 2}{2} = \frac{2}{2} = 1 \text{ units/sec}$$

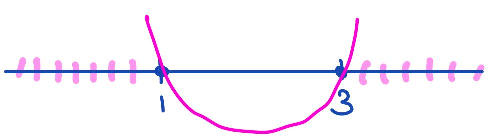
d. When is the particle at rest?

$d'(t) = 0$
$3t^2 - 12t + 9 = 0$
$3(t^2 - 4t + 3) = 0$
$3(t-3)(t-1) = 0$

$t = 3$ sec $\quad t = 1$ sec.

e. When is the particle moving forward?

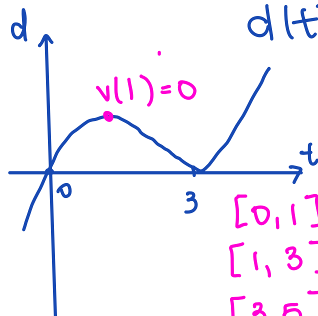
$d'(t) > 0$



$t < 1$ and $t > 3$

$v(t) = 3t^2 - 12t + 9$

f. Find the total distance traveled by the particle during the first 5 seconds.



$d(t) = t^3 - 6t^2 + 9t$
$= t(t^2 - 6t + 9)$
$= t(t-3)^2$

[0, 1]	$d(0) = 0$, $d(1) = 4$	$\lvert 4 - 0 \rvert = 4$ units
[1, 3]	$d(1) = 4$, $d(3) = 0$	$\lvert 0 - 4 \rvert = 4$ units
[3, 5]	$d(3) = 0$, $d(5) = 20$	$\lvert 20 - 0 \rvert = 20$ units

$\Rightarrow$ 28 units total

g. Find the acceleration at time t and after 4 seconds.

$v'(t) = 6t - 12$
$= a(t)$

$a(4) = 6(4) - 12$
$= 12$ units/sec^2

h. When is the particle speeding up? When is it slowing down?

$\hookrightarrow a(t) > 0$

$\hookrightarrow a(t) < 0$

$a(t) > 0 \qquad 6t - 12 > 0$
$6t > 12$
$t > 2 \qquad (2, \infty)$

$a(t) < 0 \qquad 6t - 12 < 0$
$6t < 12$
$t < 2 \qquad [0, 2)$

question

No negative time!