Name: ___________________________ Date: ____________________

Learning Goal 3.3	Creating confidence in (baby) word problems.

More Questions

1. The volume of a growing spherical cell is $V=\frac{4}{3}\pi r^3$, where the radius r is measured in μm.
   a. Find the average rate of change of V with respect to r, when r changes from 5 to 8μm.

$$\begin{aligned}\frac{\Delta V}{\Delta r} &= \frac{V(8)-V(5)}{8-5}\\ &= \frac{\frac{4}{3}\pi(8^3-5^3)}{3}\\ &= \frac{4\pi(512-125)}{9}\\ &= \frac{1\,548\pi}{9}\\ &= 172\pi\\ &\approx 540.35\ \frac{\mu\text{m}^3}{\mu\text{m}}\end{aligned}$$

   b. Find the instantaneous rate of change of V with respect to r when $r = 5\mu$m.

$$\frac{dV}{dr} = V'(r) = \frac{4}{3}\pi \times 3r^2 = 4\pi r^2$$

$$\begin{aligned}V'(5) &= 4\pi(5)^2\\ &= 100\pi\\ &\approx 314.16\ \frac{\mu\text{m}^3}{\mu\text{m}}\end{aligned}$$

2. Suppose a company has estimated that the cost of producing x items is

$$C(x) = 10\,000 + 5x + 0.01x^2.$$

   a. Find the average cost per item for producing 500 items.

$$\begin{aligned}\frac{\Delta C}{\Delta x} &= \frac{C(500)-C(0)}{500-0}\\ &= \frac{(10\,000+5(500)+0.01(500)^2)-10\,000}{500}\\ &= \frac{5(500)+0.01(500)^2}{500}\\ &= \frac{2\,500+2\,500}{500}\\ &= \$10/_{\text{item}}\end{aligned}$$

b. Find the marginal cost (instantaneous cost) at the production level of 500 items.

$$C'(x) = 5 + 0.02x$$

$$\begin{aligned} C'(500) &= 5 + 0.02(500) \\ &= 5 + 10 \\ &= \$15/\text{item} \end{aligned}$$

So to make one more item, it will cost \$15.

c. Find the actual change in cost of producing 500 to 501 items.

$$\begin{aligned} \Delta C &= C(501) - C(500) \\ &= (10\,000 + 5(501) + 0.01(501)^2) - (10\,000 + 5(500) + 0.01(500)^2) \\ &= (5(501) + 0.01(501)^2) - (5(500) + 0.01(500)^2) \\ &= (5(501 - 500) + 0.01((501)^2 - (500)^2) \\ &= 5 + 0.01(251\,001 - 250\,000) \\ &= 5 + 0.01(1\,001) \\ &= \$15.01 \end{aligned}$$

3. If a tank holds 5 000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V = 5\,000\left(1 - \frac{t}{40}\right)^2 \quad 0 \le t \le 40$$

a. Find the rate at which water is draining from the tank after

i. 5 minutes

$$\begin{aligned} V'(t) &= 10\,000\left(1 - \frac{t}{40}\right) \times -\frac{1}{40} \\ &= -250\left(1 - \frac{t}{40}\right) \end{aligned}$$

$$\begin{aligned} V'(5) &= -250\left(1 - \frac{5}{40}\right) \\ &= -250\left(\frac{7}{8}\right) \\ &\approx 219\ \text{gallons}/\text{minute} \end{aligned}$$

ii. 10 minutes

$$\begin{aligned} V'(10) &= -250\left(1 - \frac{10}{40}\right) \\ &= -250\left(\frac{3}{4}\right) \\ &\approx 188\ \text{gallons}/\text{minute} \end{aligned}$$

iii. 20 minutes

$$\begin{aligned} V'(20) &= -250\left(1-\frac{20}{40}\right) \\ &= -250\left(\frac{1}{2}\right) \\ &\approx 125 \ {}^{\text{gallons}}/_{\text{minute}} \end{aligned}$$

iv. 40 minutes

$$\begin{aligned} V'(10) &= -250\left(1-\frac{40}{40}\right) \\ &= -250(0) \\ &\approx 0 \ {}^{\text{gallons}}/_{\text{minute}} \end{aligned}$$

b. At what time is the water flowing out the fastest? The slowest?

$$V'(t) = -250\left(1-\frac{t}{40}\right)$$

With a linear relationship with a negative slope, the water flows the fastest at the beginning (the fraction is quite small and so you're not taking much away) and the slowest at the end (the fraction is quite large so there's not a lot left).

4. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 ${}^{\text{cm}}/_{\text{s}}$. Find the rate at which the area within the circle is increasing after

a. 1 second

Two equations are stated here:

$r = 60\,t$	$A = \pi r^2$
the rate of growth of the radius	the area of the circle

Combined, our function is

$$\begin{aligned} A(t) &= \pi(60t)^2 \\ &= 3\,600\pi t^2 \end{aligned}$$

$$\begin{aligned} A'(t) &= 7200\pi t \\ A'(1) &= 7200\pi(1) \\ &\approx 22\,619 \ {}^{\text{cm}^2}/_{\text{s}} \end{aligned}$$

b. 3 seconds

$$\begin{aligned} A'(3) &= 7200\pi(3) \\ &\approx 67\,858 \ {}^{\text{cm}^2}/_{\text{s}} \end{aligned}$$

c. 5 seconds

$$\begin{aligned} A'(5) &= 7200\pi(5) \\ &\approx 113\,097 \ {}^{\text{cm}^2}/_{\text{s}} \end{aligned}$$