Name: _____ Date: _____

Recall the Product of Powers rule to simplify the following expressions.

1. $3^6 3^2$

$= 3^{6+2}$

$= 3^8$

2. $6^7 6^2$

$= 6^{7+2}$

$= 6^9$

3. $7^6 7^1$

$= 7^{6+1}$

$= 7^7$

And the Quotient of Powers rule:

4. $3^6 \div 3^2$

$= \frac{3\times3\times3\times3\times3\times3}{3\times3}$

$= 3^4$

$= 3^{6-2}$

$= 3^4$

5. $\frac{6^2}{6^7} =$

$= 6^{2-7}$

$= 6^{-5}$

6. $7 \div 7^6$

$= 7^{1-6}$

$= 7^{-5}$

$= \frac{7}{1}$

$= \frac{7}{7\times7\times7\times7\times7\times7}$

$= \frac{1}{7^5}$

Take a (silent) minute. What do you think the negative exponents represent?

the reciprocal of the base raised to the positive of the exponent.

When *x* is any non-zero number and *n* is a rational number,

$$x^{-n} = \frac{1}{x^n}, \quad x \neq 0$$

$5 \div 15 = \frac{3}{2}$
$5 \div 10$

Example Evaluate the following expressions **without a calculator.** Leave your answers as fractions.

1. 7^{-2} $= \left(\frac{1}{7}\right)^2$
$= \frac{1}{7^2} = \frac{1}{49}$

2. $(-1.5)^{-3}$ $= \left(-\frac{3}{2}\right)^{-3}$
$\left(-\frac{1}{1.5}\right)^3$ $= \left(-\frac{2}{3}\right)^3 = \frac{(-2)^3}{3^3}$
$= -\frac{8}{27}$

3. $(-\frac{3}{4})^{-3}$ $= \left(-\frac{4}{3}\right)^3$
$= -\frac{64}{27}$

4. $(\frac{10}{3})^{-2}$
$= \frac{9}{100}$
$= \left(\frac{3}{10}\right)^2$

What if the exponent is not in integer? Take a minute (silently) to consider.

$$8^{-2/3}$$

$\left(\frac{1}{8}\right)^{2/3}$

$\left(\sqrt[3]{\frac{1}{8}}\right)^2 = \frac{1}{4}$

$\left(\frac{1}{2}\right)^2$
$= \frac{1}{4}$

$\left(\frac{1}{8}\right)^{2/3}$

$\sqrt[3]{\left(\frac{1}{8}\right)^2} = \frac{1}{4}$

$\sqrt[3]{\frac{1}{64}}$
$= \frac{1}{4}$

Example Evaluate the following expressions, **without using a calculator**.

1. $16^{-5/4}$
$= \left(\frac{1}{16}\right)^{5/4}$
$= \left(\sqrt[4]{\frac{1}{16}}\right)^5$
$= \left(\frac{1}{2}\right)^5 = \frac{1}{2^5} = \frac{1}{32}$

2. $(\frac{25}{36})^{-1/2}$
$= \left(\frac{36}{25}\right)^{\frac{1}{2}}$
$= \sqrt{\frac{36}{25}}$
$= \frac{6}{5}$

3. $\left(\frac{9}{16}\right)^{-3/2}$

$= \frac{64}{27}$

4. $(-0.008)^{-4/3}$

$= \left(\frac{-8}{1000}\right)^{-\frac{4}{3}}$

$= \left(\frac{1000}{-8}\right)^{\frac{4}{3}}$

$= \left(\sqrt[3]{\frac{-1000}{8}}\right)^{4}$

$= \left(\frac{-10}{2}\right)^{4}$

$= 5^4$

$= 10000$

$= 625$

Example Paleontologists use measurements from fossilized dinosaur tracks and the formula

$$v = 0.155 s^{5/3} f^{-7/6}$$

to estimate the speed at which the dinosaur travelled. In the formula, *v* is the speed in metres per second, *s* is the distance between successive footprints of the same foot, and *f* is the foot length in metres. If *s* = 1.5 m and *f* = 0.3 m, find the estimate the speed of the dinosaur.