Name: _____ Date: _____

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

More Questions – Solutions

1. Solve each equation algebraically over the domain $0 \le x < 2\pi$.

a. $4\sin^2 x - 1 = 0$

$$(2\sin x + 1)(2\sin x - 1) = 0$$

$2\sin x + 1 = 0$

$2\sin x = -1$

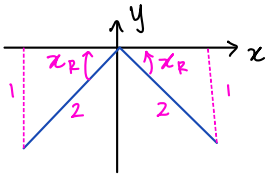
$\sin x = -\frac{1}{2}$

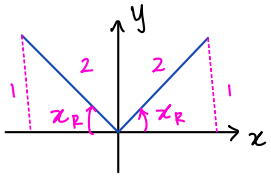
$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x_R = \frac{\pi}{6}$





$x_1 = \pi + \frac{\pi}{6}$

$= \frac{7\pi}{6}$

$x_2 = 2\pi - \frac{\pi}{6}$

$= \frac{11\pi}{6}$

$x_3 = \frac{\pi}{6}$

$x_4 = \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

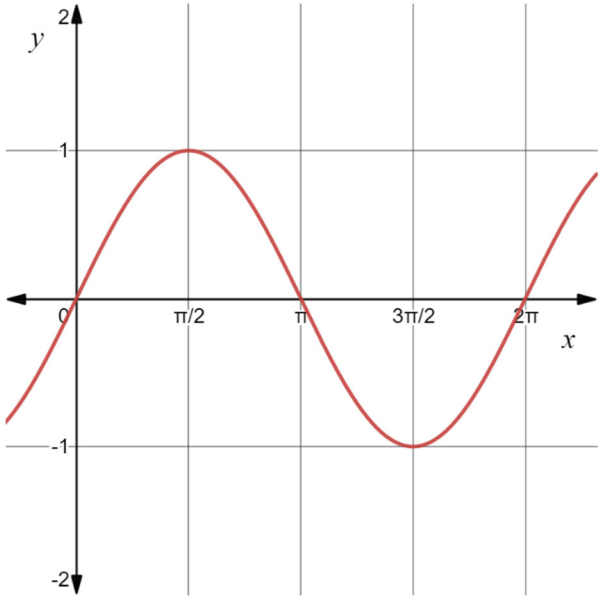
b. $\sin^2 x - \sin x = 0$

$$\sin x\,(\sin x - 1) = 0$$

$\sin x = 0$

$\sin x - 1 = 0$

$\sin x = 1$



$x_1 = 0$

$x_2 = \pi$

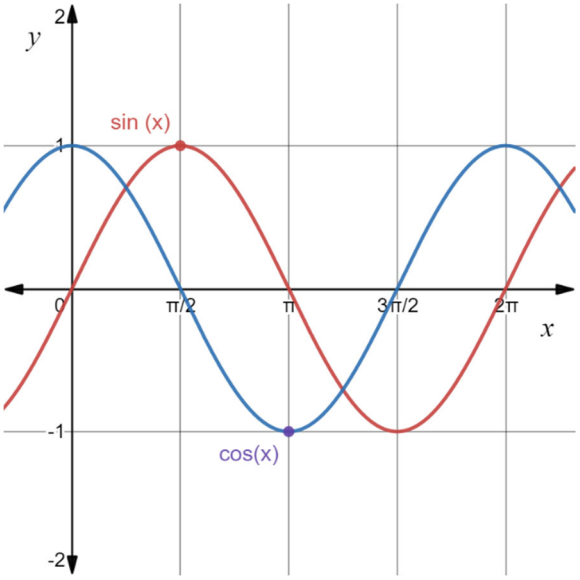
$x_3 = \frac{\pi}{2}$

c. $\sin 2x - 1 = \cos 2x$

$$2\sin x\cos x - 1 = 2\cos^2 x - 1$$
$$2\sin x\cos x = 2\cos^2 x$$
$$2\sin x\cos x - 2\cos^2 x = 0$$
$$2\cos x(\sin x - \cos x) = 0$$

$2\cos x = 0$ $\qquad$ $\sin x - \cos x = 0$

$\cos x = 0$ $\qquad$ $\sin x = \cos x$

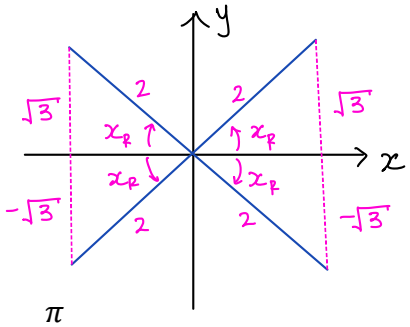


$x_1 = \frac{\pi}{2}$ $\qquad$ $x_1 = \frac{\pi}{4}$

$x_2 = \frac{3\pi}{2}$ $\qquad$ $x_2 = \frac{5\pi}{4}$

d. $2\sin^2 x - 3\cos 2x = 3$

$$2\sin^2 x - 3(1 - 2\sin^2 x) = 3$$
$$2\sin^2 x - 3 + 6\sin^2 x = 3$$
$$8\sin^2 x = 6$$
$$\sin^2 x = \frac{3}{4}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x_R = \frac{\pi}{3}$$



$x_1 = \frac{\pi}{3}$ $\qquad$ $x_3 = \pi + \frac{\pi}{3}$

$= \frac{4\pi}{3}$

$x_2 = \pi - \frac{\pi}{3}$ $\qquad$ $x_4 = 2\pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$ $\qquad$ $= \frac{5\pi}{3}$

e. $\cos x + 1 = 2\sin^2 x$

$$\cos x + 1 = 2(1 - \cos^2 x)$$
$$\cos x + 1 = 2 - 2\cos^2 x$$
$$2\cos^2 x + \cos x - 1 = 0$$
$$2\cos^2 x + \cos x - 1 = 0$$
$$(2\cos x - 1)(\cos x + 1) = 0$$

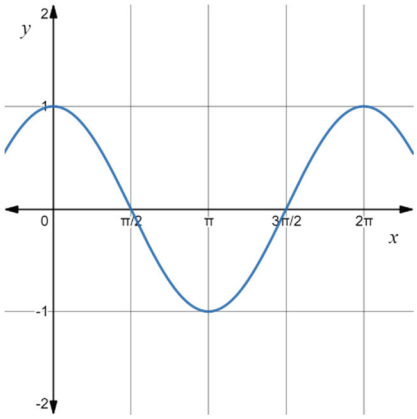
$2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$\cos x + 1 = 0$

$\cos x = -1$



$x_1 = x_R$

$= \frac{\pi}{3}$

$x_3 = \pi$

$x_2 = 2\pi - \frac{\pi}{3}$

$= \frac{5\pi}{3}$

f. $\sec^2 x + 5\sec x + 6 = 0$

$$(\sec x + 2)(\sec x + 3) = 0$$

$\sec x + 2 = 0$

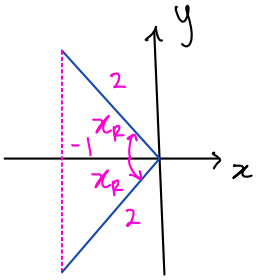
$\sec x = -2$

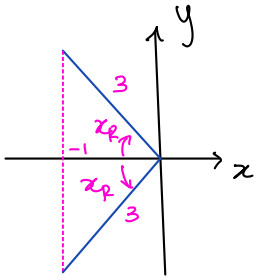
$\cos x = -\frac{1}{2}$

$\sec x + 3 = 0$

$\sec x = -3$

$\cos x = -\frac{1}{3}$





$x_R = \frac{\pi}{3}$

$x_2 = \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$x_2 = \pi + \frac{\pi}{3}$

$= \frac{4\pi}{3}$

$x_R \approx 1.23$

$x_3 \approx \pi - 1.23$

≈ 1.91

$x_4 \approx \pi + 1.23$

≈ 4.37