

Name: _____

Date: _____

Learning Goal 7.1	Applying one or more transformations to an exponential function, including translations, stretches and reflections.
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An exponential function has an equation of the form $f(x) = Ab^x$, where A and b are constants and $b > 0$.

Example The population of a bacterial culture triples every hour. When the scientist observed the culture, it had already been growing for some time. She developed the equation for the population, P , after t hours as $P = 700(3)^t$, based on $t = 0$ representing the time she started her measurements. How many bacterial cells were there 2 hours before she started measuring?

$\hookrightarrow t = -2$

$$700 \times 3^0$$

$$= 700 \times 1$$

$$= 700$$

$$P = 700 \times 3^t$$

initial \uparrow
POPULATION

\uparrow SHOWING THE POPULATION IS TRIP LING

$$P = 700 \times 3^{-2}$$

$$= 700 \times \frac{1}{9}$$

$$= 78 \text{ Bacteria}$$

Example The half-life of a radioactive element can be modelled by

$$M = M_0 \left(\frac{1}{32}\right)^{t/45}$$

where M_0 is the initial mass of the element; t is the elapsed time, in hours and M is the mass that remains after time t . Determine the half-life of the element.

\uparrow HOW LONG IT TAKES FOR HALF THE MASS TO ... DISAPPEAR?

Need to make this a 2!

$$\left(\frac{1}{32}\right)^{t/45}$$

$$= \left(\frac{1}{2^5}\right)^{t/45}$$

$$= \left(\frac{1}{2}\right)^{\frac{5 \times t}{45}}$$

$$\frac{5 \times t}{45}$$

$$= \frac{t}{9}$$

THE HALF LIFE IS 9 HOURS.

Example Cobalt-60, which has a half-life of 5.3 years is used in medical radiology. A sample of 60 mg of the material is present today.

- a. Write an equation to relate the amount of cobalt-60 remaining and the number of half-life periods.

$$A = 60 \left(\frac{1}{2} \right)^{t/5.3}$$

initial quantity half-life HALF-LIFE IS 5.3 YEARS.

- b. What amount will be present in 10.6 years?

$$\frac{10.6}{5.3} = 2 \Rightarrow 2 \text{ HALF LIVES HAVE BEEN COMPLETED.}$$

$$\begin{aligned} A &= 60 \left(\frac{1}{2} \right)^2 \\ &= 60 \left(\frac{1}{4} \right) = 25 \text{ mg} \end{aligned}$$

- c. How many years will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount?

$$12.5\% = \frac{12.5}{100} = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$$

$$\frac{1}{8} \text{ OF } 60 = 7.5 \text{ mg}$$

$$\frac{7.5}{60} = \frac{60}{60} \left(\frac{1}{2} \right)^{t/5.3}$$

$$\frac{1}{8} = \left(\frac{1}{2} \right)^{t/5.3}$$

$$\left(\frac{1}{2} \right)^3 = \left(\frac{1}{2} \right)^{t/5.3}$$

$$3 = \frac{t}{5.3}$$

$$t = 15.9 \text{ years.}$$