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Chapter 3b Review
Derivatives

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 3.3	Using more derivative rules.

1. Find the following derivatives.

Developing		
a. $y = \log_4(1+3x)$ $\frac{dy}{dx} = \frac{3}{(1+3x)\ln 4}$	b. $y = \ln\lvert\sec 5x + \tan 5x\rvert$ $\frac{dy}{dx} = 5\sec 5x$	c. $x^2 + y^2 = 25$ $\frac{dy}{dx} = -\frac{x}{y}$
d. $9x^2 - 16y^2 = -144$ $\frac{dy}{dx} = \frac{9x}{16y}$	e. $15y^2 = 2x^3$ $\frac{dy}{dx} = \frac{x^2}{5y}$	f. $\frac{x^2}{16} + \frac{3y^2}{13} = 1$ $\frac{dy}{dx} = -\frac{13x}{48y}$
g. $4x^2 + 9y^2 = 36$ $\frac{dy}{dx} = -\frac{4x}{9y}$	h. $\frac{1}{x} + \frac{1}{y} = 1$ $\frac{dy}{dx} = -\frac{y^2}{x^2}$	i. $\sqrt{x} + \sqrt{y} = 4$ $\frac{dy}{dx} = -\frac{\sqrt{xy}}{x}$
j. $x^2 - y^2 = 1$ $\frac{dy}{dx} = \frac{x}{y}$	k.	l.
Proficient		
a. $xe^y = x - y$ $\frac{dy}{dx} = \frac{1-e^y}{1+xe^y}$	b. $\cos(xy) = 1 + \sin y$ $\frac{dy}{dx} = -\frac{y\sin(xy)}{x\sin(xy) + \cos y}$	c. $y = (x^2+2)^2(x^4+4)^4$ $\frac{dy}{dx} = 4x(x^2+2)(x^4+4)^3(5x^4 + 8x^2 + 4)$
d. $2xy - y^3 = 4$ $\frac{dy}{dx} = -\frac{2y}{2x - 3y^2}$	e. $3xy^2 + y^3 = 8$ $\frac{dy}{dx} = -\frac{y}{2x+y}$	f. $x^2 + y^2 + 5y = 10$ $\frac{dy}{dx} = -\frac{2x}{2y+5}$
g. $x^3 + y^3 - 3xy = 17$ $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$	h. $5x^2 - 6xy + 5y^2 = 16$ $\frac{dy}{dx} = \frac{3y - 5x}{5y - 3x}$	i. $(x+y)^3 = 12x$ $\frac{dy}{dx} = \frac{4}{(x+y)^2} - 1$
j. $\sqrt{x+y} - 2x = 1$ $\frac{dy}{dx} = 4\sqrt{x+y} - 1$	k. $4x^2y - 3y = x^3$ $\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$	l. $x^3 + y^3 = 6xy$ $\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$

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m. $xy + 2x + 3x^2 = 4$ $\frac{dy}{dx} = -\frac{6x + y + 2}{x}$	n. $4x^2y - 3y = x^3$ $\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$	o. $1 + y + x^2\,y^3 = 0$ $\frac{dy}{dx} = -\frac{2xy^3}{1 + 3x^2y^2}$
p. $\sqrt{x+y} = 1 + x^2y^2$ $\frac{dy}{dx} = \frac{1 - 4xy^2\sqrt{x+y}}{4x^2y\sqrt{x+y} - 1}$	q. $x^2 - 2xy + y^3 = c$ $\frac{dy}{dx} = \frac{2(y-x)}{3y^2 - 2x}$	r. $x^2y + xy^2 = 3x$ $\frac{dy}{dx} = \frac{-2xy - y^2 + 3}{x(x + 2y)}$
s. $y^5 + x^2y^3 = 1 + ye^{x^2}$ $\frac{dy}{dx} = \frac{2xy(y^2 - e^{x^2})}{-3x^2y^2 + e^{x^2} - 5y^4}$	t. $x^2y^2 + x\sin y = 4$ $\frac{dy}{dx} = \frac{-2xy^2 - \sin y}{x(2xy + \cos y)}$	u. $1 + x = \sin(xy^2)$ $\frac{dy}{dx} = \frac{(1 - y^2\cos(xy^2))\sec(xy^2)}{2xy}$
v. $4\cos x \sin y = 1$ $\frac{dy}{dx} = \tan x \tan y$	w. $y\sin x^2 = x\sin y^2$ $\frac{dy}{dx} = \frac{2xy\cos x^2 - \sin y^2}{2xy\cos y^2 - \sin x^2}$	x.

Extending	
a. $y = (\sin x)^{\ln x}$ $\frac{dy}{dx} = (\sin x)^{\ln x}\left(\frac{\ln(\sin x)}{x} + \ln x \cot x\right)$	b. $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 10$ $\frac{dy}{dx} = \frac{y}{x}$
c. $e^{x^2y} = x + y$ $\frac{dy}{dx} = \frac{1 - 2xye^{x^2y}}{x^2e^{x^2y} - 1}$	d. $\sqrt{xy} = 1 + x^2y$ $\frac{dy}{dx} = \frac{y - 4xy\sqrt{xy}}{x(2x\sqrt{xy} - 1)}$
e. $\tan(x - y) = \frac{y}{1 + x^2}$ I am sorry – if you work through this …. you're amazing 😊 $\frac{dy}{dx} = \frac{x^4(-\sec^2(x-y)) - 2x^2\sec^2(x-y) - 2xy - \sec^2(x-y)}{(x^2+1)(-x^2\sec^2(x-y) - \sec^2(x-y) - 1)}$	f. $xy = \cot(xy)$ $\frac{dy}{dx} = -\frac{y}{x}$
g. $\sin x + \cos y = \sin x \cos y$ $\frac{dy}{dx} = \frac{\csc y\,(\cos x \cos y - \cos x)}{\sin x - 1}$	h. $\sqrt{xy} = 1 + x^2y$ $\frac{dy}{dx} = \frac{y - 4xy\sqrt{xy}}{x(2x\sqrt{xy} - 1)}$

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2. Find the derivative in question.

Developing	
a. $f(x) = \sqrt{x}$ $f''(x)$ $f''(x) = \frac{\sqrt{x}}{4x^2}$	b. $y = x^{10} + 3x^6$ $\frac{d^3y}{dx^3}$ $\frac{d^3y}{dx^3} = 360x^3(2x^4 + 1)$
c. $f(x) = x^2 + \frac{1}{x^2}$ $f'''(x)$ $f'''(x) = \frac{2(x^4 + 3)}{x^4}$	d. $h(x) = 3x^4 - 4x^3 - 3x^2 - 5$ $h''(x)$ $h''(x) = 6(6x^2 - 4x - 1)$
e. $y = 4x^{3/2} - x^{-2}$ $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = \frac{3(x^3\sqrt{x} - 2)}{x^4}$	f. $h(x) = \sqrt[3]{x^5}$ $h''(x)$ $h''(x) = \frac{10\sqrt[3]{x^2}}{9x}$
Proficient	
a. $f(x) = \frac{x}{1+x}$ $f''(x)$ $f''(x) = -\frac{2}{(1+x)^3}$	b. $f(x) = \frac{2x}{x+1}$ $f''(x)$ $f''(x) = -\frac{4}{(x+1)^3}$
c. $y = (1-x)^2$ $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 2$	d. $g(x) = \sqrt{3x - 6}$ $g'''(x)$ $g'''(x) = \frac{81\sqrt{3x - 6}}{8(3x - 6)^3}$
e. $y = (2x + 4)^3$ $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 48(x + 2)$	f.

3. Find an equation of the tangent line to the curve at the given point.

Proficient	
a. $y = x^2 \ln x$ $(1, 0)$ $y = x - 1$	b. $f(x) = \frac{1}{(1+x)^2}$ $\left(1, \frac{1}{2}\right)$ $y - \frac{1}{2} = -\frac{1}{4}(x - 1)$

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Proficient
4. Let $f(x) = \log_c(3x^2 - 2)$. For what value of c is $f'(1) = 3$? $c = e^2$
5. At what point is the tangent to the curve $x + y^2 = 1$ parallel to the line $x + 2y = 0$? $(0, 1)$
6. Determine the equation of the normal at $(1, -1)$ to the curve $y^2 = \frac{x^3}{2 - x}$ $y + 1 = \frac{1}{2}(x - 1)$

7. Find a general expression for $f^{(n)}(x)$ given

Extending		
a. $f(x) = xe^x$ $f^{(n)}(x) = ne^x + xe^x$	b.	c.

8. Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

Extending	
a. $f(x) = \sin 3x$ $\quad f^{(100)}(x)$ $f^{(100)}(x) = 3^{100} \sin 3x$	b. $f(x) = xe^{-x}$ $\quad f^{(1\,000)}(x)$ $f^{(1\,000)}(x) = -1\,000e^{-x} + xe^{-x}$

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Derivatives

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Learning Goal 3.4	Creating confidence in (baby) word problems.

1. An object is moving along a straight line. It's position, $s(t)$, to the right of a fixed point is given by the graph shown. When is the object moving to the right, to the left and at rest?

$$t \in (0,2) \cup (6,\infty) \quad \text{right}$$

$$t \in (2,6) \quad \text{left}$$

$$t = 2, 6 \quad \text{at rest}$$

2. Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is $F = \frac{GmM}{r^2}$ where G is the gravitational constant and r is the distance between the bodies.
   a. Find dF/dr and explain its meaning. What does the negative sign indicate?

$$\frac{dF}{dr} = -\frac{2GmM}{r^3}$$

   b. Suppose it is known that the earth attracts an object with a force that decreases at the rate of $2\ \text{N}/\text{km}$ when $r = 20\ 000$ km. How fast does this force change when $r = 10\ 000$ km?

$$\frac{dF}{dr} = -16\ \text{N}/\text{km}$$

3. A particle moves according to the law of motion $s(t) = 0.01t^4 - 0.04t^3, t \geq 0$ (t is in seconds and s is in feet).
   a. Find the velocity at time t.

$$v(t) = 0.04t^2(t-3)$$

   b. When is the particle at rest?

$$t = 0, 3$$

   c. Find the total distance traveled during the first 8 seconds.

$$d_t = 21.02 \text{ feet}$$

d. Find the acceleration at time t.

$$a(t) = 0.12t(t-2)$$

e. When is the particle speeding? When is it slowing down?

$$[0,2] \cup [3,\infty) \text{ accelerating}$$

$$[2,3] \text{ slowing}$$

4. The position of an object moving on a line is given by $s(t) = 6t^2 - t^3$ where s is in metres and t is in seconds.
   a. Determine the velocity and acceleration of the object at $t = 2$.

$$v(2) = 12 \ {}^{\text{m}}/_{\text{s}}$$

$$a(2) = 0 \ {}^{\text{m}}/_{\text{s}^2}$$

   b. At what time(s) is the object at rest?

$$t = 0, 4$$

   c. In which direction is the object moving at $t = 5$?

Negative

   d. When is the object moving in a positive direction?

$$0 < t < 4$$

   e. When does the object return to its initial position?

$$t = 6 \text{ s}$$

5. Discuss the motion of an object moving on a horizontal line if its position is given by $s(t) = t^2 - 10t, 0 \leq t \leq 12$, where s is in metres and t is in seconds. Include initial velocity, final velocity and any acceleration in your discussion.

$$v(0) = -10 \ {}^{\text{m}}/_{\text{s}}$$

$$v(12) = 14 \ {}^{\text{m}}/_{\text{s}}$$

6. A baseball is hit vertically upward. The position function $s(t)$, in metres, of the ball above the ground is $s(t) = -5t^2 + 30t + 1$, where t is in seconds.
   a. Determine the maximum height reached by the ball.

$$46 \text{ metres}$$

b. Determine the velocity of the ball when it is caught 1 metre above the ground.

$$v(6) = -30\ {}^{\text{m}}\!/_{\text{s}}$$

7. A particle moves along a straight line with the equation of motion

$$s = \frac{1}{3}t^3 - 2t^2 + 3t, t \geq 0$$

a. Determine the particle's velocity and acceleration at any time t.

$$v(t) = s'(t) = t^2 - 4t + 3$$

$$a(t) = v'(t) = s''(t) = 2t - 4$$

b. When does the motion of the particle change direction?

$$t = 1, 3$$

c. When does the particle return to its initial position?

$$t = 3$$