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Chapter 3c Review
Derivatives

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 3.5	Using the last of the derivative rules (for now).

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$	$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

1. Find the following derivatives.

Developing	
a. $f(x) = (\cot^{-1} x)^{1/3}$	b. $g(x) = \sin^{-1}(3^x)$
c. $f(x) = (x^2+1)\tan^{-1} x$	d. $y = \tan^{-1}\sqrt{x}$
e. $f(x) = \sin^{-1}(2x+1)$	f. $h(x) = \sqrt{1-x^2}\sin^{-1} x$
g. $y = (1+x^2)\tan^{-1} x$	h. $y = \tan^{-1}\left(x - \sqrt{1+x^2}\right)$
i. $h(x) = \cot^{-1} x + \cot^{-1}\left(\frac{1}{x}\right)$	j. $y = x\cos^{-1} x - \sqrt{1-x^2}$
k. $h(x) = \sqrt{1-x^2}\cos^{-1} x$	l. $f(x) = x\tan^{-1}(4x)$
Proficient	
a. $\sin^{-1}(xy) + xy = x$	b. $\tan^{-1}(x-y) = xy$
c. $y = \cos^{-1}(e^{2x})$	d. $y = \tan^{-1}(\cos x)$
e. $y = \tan^{-1}(\sin x)$	f. $y = \tan^{-1}(\cos x)$
g. $y = \tan^{-1}(\sin\sqrt{x})$	h.

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Learning Goal 3.6	Linear approximations.

1. Determine the linear approximation $L(x)$ of each function around the given x value.

		Developing			
a.	$f(x)=\sqrt{x}$	$x=3$	b.	$f(x)=\frac{1}{x}$	$x=5.3$
c.	$f(x)=\frac{1}{x^2}$	$x=2.8$	d.	$f(x)=x^2+3$	$x=2.2$
e.	$f(x)=(x-2)^3$	$x=3.1$	f.	$f(x)=x^2+3$	$x=2.2$
		Proficient			
a.	$f(x)=\ln(1+x)$	$x=0.1$	b.	$f(x)=\sqrt[3]{1+3x}$	$x=0.01$
c.	$f(x)=\sqrt{25-x^2}$	$x=3.1$	d.	$f(x)=\sqrt[3]{1+3x}$	$x=0.01$

2. For what values of x is the linear approximation accurate to within 0.1?

		Extending			
a.	$f(x)=\sqrt{x}$	$x=3$	b.	$f(x)=\frac{1}{x}$	$x=5.3$
c.	$f(x)=\frac{1}{x^2}$	$x=2.8$	d.	$f(x)=x^2+3$	$x=2.2$
e.	$f(x)=(x-2)^3$	$x=3.1$	f.	$f(x)=x^2+3$	$x=2.2$
g.	$f(x)=\ln(1+x)$	$x=0.1$	h.	$f(x)=\sqrt[3]{1+3x}$	$x=0.01$
i.	$f(x)=\sqrt{25-x^2}$	$x=3.1$	j.	$f(x)=\sqrt[3]{1+3x}$	$x=0.01$

2. Use linear approximation to estimate the following values.

	Developing		
a.	$(1.9)^3$	b.	$\sqrt{0.041}$

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Learning Goal 3.7	Creating confidence in word problems.

Volume of Prisms	Volume of a Sphere	Volume of a Pyramid
$V = A_{\text{base}} \times h$	$V = \frac{4}{3}\pi r^3$	$V = \frac{1}{3}A_{\text{base}} \times h$

1. Suppose the quantity demanded weekly of a product is related to its unit price by the equation

$$p + q^2 = 144$$

   where p is measured in dollars and q is measured in units of a thousand. What is the rate of change of the quantity demanded when $q = 9, p = 63$ and the unit price is increasing at a rate of \$2 per week?

2. The demand equation for a certain product is $100q^2 + 9p^2 = 3\ 600$ where$q$ is the number (in thousands) of units demanded each week when the unit price is \$$p$. What is the rate of change of the quantity demanded when the unit price is \$14 and the selling price is dropping at a rate of \$0.15 per unit, per week.
3. Air is being pumped into a spherical balloon at a constant rate of 3 $^{\text{cm}^3}/_{\text{s}}$. How fast is the radius of the balloon increasing when the radius reaches 5 cm?
4. A cylindrical tank standing upright (with one circular base on the ground) has radius 20 cm. How fast does the water level in the tank drop when the water is being drained at 25 $^{\text{cm}^3}/_{\text{s}}$
5. A ladder 13 metres long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate 0.6 $^{\text{m}}/_{\text{s}}$. How fast is the top sliding down the wall when the foot of the ladder is 5 m from the wall?
6. A rotating beacon is located 2 miles out in the water. Let A be the point on the shore that is closest to the beacon. As the beacon rotates at 10 $^{\text{rev}}/_{\text{min}}$, the beam of light sweeps down the shore once each time it revolves. Assume that the shore is straight. How fast is the point where the beam hits the shore moving at an instant when the beam is lighting up a point 2 miles along the shore from point A?
7. A baseball diamond is a square 90 ft on a side. A player runs from first base to second at 15 $^{\text{ft}}/_{\text{s}}$. At what rate is the player's distance from the third base decreasing when she is half way from first to second base?
8. Sand is poured onto a surface at 15 $^{\text{cm}^3}/_{\text{s}}$, forming a conical pile whose base diameter is always equal to its altitude. How fast is the altitude of the pole increasing when the pole is 3 cm high?

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9. Each edge of a cube is expanding at a rate of $4\ {}^{\mathrm{cm}}/_{\mathrm{s}}$. How fast is the volume changing when each edge is $5\ \mathrm{cm}$? At what rate is the surface area changing when each edge is $7\ \mathrm{cm}$?
10. Oil that is spilled from a ruptured tanker spreads in a circle. The area of the circle increases at a constant rate of $6\ {}^{\mathrm{km}^2}/_{\mathrm{h}}$. How fast is the radius of the spill increasing when the area is $9\pi\ \mathrm{km}^2$?
11. How fast must someone let out line if a kite is $30\ \mathrm{m}$ high, $40\ \mathrm{m}$ away horizontally, and continuing to move away horizontally at a rate of $10\ {}^{\mathrm{m}}/_{\mathrm{min}}$?
12. Two cyclists depart at the same time from a starting point along routes that make an angle of ${}^{\pi}/_{3}$ radians with each other. The first cyclist is travelling at $15\ {}^{\mathrm{km}}/_{\mathrm{h}}$, while the second cyclist is moving at $20\ {}^{\mathrm{km}}/_{\mathrm{h}}$. How fast are the two cyclists moving apart after 2 hours?
13. A railroad bridge is $20\ \mathrm{m}$ above, and at right angles to, a river. A person in a train travelling at $60\ {}^{\mathrm{km}}/_{\mathrm{h}}$ passes over the centre of the bridge at the same instant that a person in a motorboat travelling $20\ {}^{\mathrm{km}}/_{\mathrm{h}}$ passes under the centre of the bridge. How fast are the two people separating $10\ \mathrm{s}$ later.