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Chapter 4 Review

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 4.1	Examining angles in standard position in both radians and degrees. Exploring the unit circle, reference and coterminal angles and special angles.

Developing

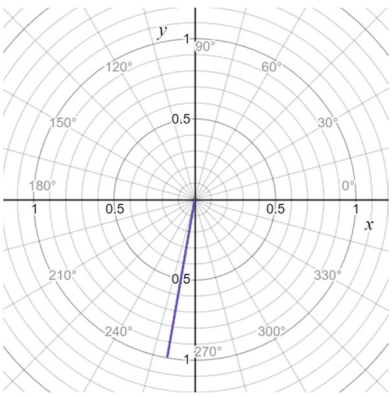
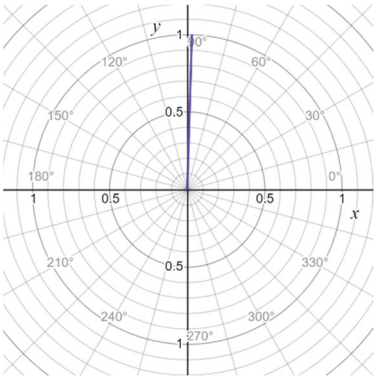
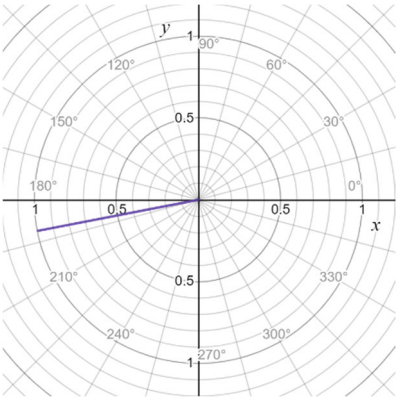
1. Convert to radians, leave your answer as an exact value in lowest terms.

a. $60°$ $= \frac{\pi}{3}$	b. $150°$ $= \frac{5\pi}{6}$	c. $570°$ $= \frac{19\pi}{6}$
d. $-225°$ $= -\frac{5\pi}{4}$	e. $680°$ $= \frac{34\pi}{9}$	f. $-450°$ $= -\frac{5\pi}{2}$

2. Convert to degrees, round your answer to the nearest degree.

a. $\frac{7\pi}{4}$ $= 315°$	b. $\frac{10\pi}{3}$ $= 600°$	c. $\frac{5\pi}{6}$ $= 150°$
d. 30 $\approx 1719°$	e. $\frac{-2\pi}{7}$ $\approx -51°$	f. -1 $\approx -57°$

3. Find the reference angle of each angle and sketch the angle in standard position. Write a general formula for coterminal angles.

a. $280°$ $\theta_R = 80°$  $\theta = 280° + 360n, n \in \mathbb{Z}$	b. $88°$ $\theta_R = 88°$  $\theta = 88° + 360n, n \in \mathbb{Z}$	c. $191°$ $\theta_R = 11°$  $\theta = 191° + 360n, n \in \mathbb{Z}$

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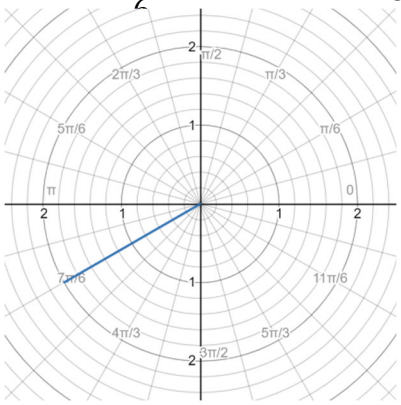
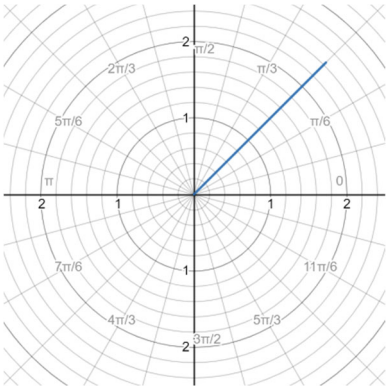
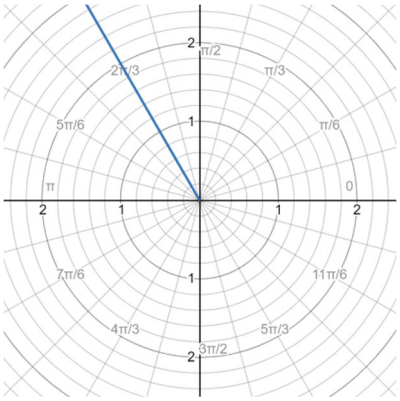
d. $-\frac{5\pi}{6}$ $\theta_R = \frac{\pi}{6}$  $\theta = \frac{7\pi}{6} + 2\pi n, n \in \mathbb{Z}$	e. $\frac{9\pi}{4}$ $\theta_R = \frac{\pi}{4}$  $\theta = \frac{\pi}{4} + 2\pi n, n \in \mathbb{Z}$	f. $\frac{8\pi}{3}$ $\theta_R = \frac{\pi}{3}$  $\theta = \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z}$

4. Determine the arc length subtended by each central angle. Give answers to the nearest hundredth of a unit.

a. $r = 20$ cm, $\theta = \frac{2\pi}{3}$ $= 41.89$ cm	b. $r = 15$ mm, $\theta = 195°$ $= 51.05$ mm	c. $r = 12$ m, $\theta = -\frac{3\pi}{4}$ $= 28.27$ m

5. Determine the area of the sector subtended by each central angle. Give answers to the nearest tenth of a square unit.

a. $r = 20$ cm, $\theta = \frac{2\pi}{3}$ $= 418.9$ cm^2	b. $r = 15$ mm, $\theta = 195°$ $= 382.9$ mm^2	c. $r = 12$ m, $\theta = -\frac{3\pi}{4}$ $= 169.6$ m^2

Proficient

6. Determine the exact values of the following trigonometric ratios. Include a diagram with your solution.

a. $\cos 330°$ $= \frac{\sqrt{3}}{2}$	b. $\sin 240°$ $= -\frac{\sqrt{3}}{2}$	c. $\tan 135°$ $= -1$
d. $\cot(-120°)$ $= \frac{\sqrt{3}}{3}$	e. $\csc(-330°)$ $= 2$	f. $\sec 315°$ $= \sqrt{2}$
g. $\cos\left(-\frac{5\pi}{3}\right)$ $= \frac{1}{2}$	h. $\sin\left(\frac{5\pi}{4}\right)$ $= -\frac{\sqrt{2}}{2}$	i. $\tan\left(\frac{5\pi}{6}\right)$ $= -\frac{\sqrt{3}}{3}$
j. $\sec\left(\frac{\pi}{6}\right)$ $= \frac{2\sqrt{3}}{3}$	k. $\cot\left(-\frac{4\pi}{3}\right)$ $= -\frac{\sqrt{3}}{3}$	l. $\csc\left(\frac{15\pi}{4}\right)$ $= -\sqrt{2}$

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Extending

7. Given the following information, find the exact value of the other five trigonometric ratios.

a. $\cos\theta = -\frac{1}{4} \quad \theta \in III$

$\sin\theta = -\frac{\sqrt{15}}{4}$

$\tan\theta = \sqrt{15}$

$\sec\theta = -4$

$\csc\theta = -\frac{4\sqrt{15}}{15}$

$\cot\theta = \frac{\sqrt{15}}{15}$

b. $\tan\theta = -\frac{3}{7} \quad \theta \in II$

$\sin\theta = \frac{3\sqrt{58}}{58}$

$\cos\theta = -\frac{7\sqrt{58}}{58}$

$\sec\theta = -\frac{\sqrt{58}}{7}$

$\csc\theta = \frac{\sqrt{58}}{3}$

$\cot\theta = -\frac{7}{3}$

c. $\sin\theta = -\frac{3}{4} \quad \theta \in IV$

$\cos\theta = \frac{\sqrt{7}}{4}$

$\tan\theta = \frac{3\sqrt{7}}{7}$

$\sec\theta = \frac{4\sqrt{7}}{7}$

$\csc\theta = -\frac{4}{3}$

$\cot\theta = \frac{\sqrt{7}}{3}$

d. $\sec\theta = \frac{3}{2} \quad \theta \in I$

$\sin\theta = \frac{\sqrt{5}}{3}$

$\cos\theta = \frac{2}{3}$

$\tan\theta = \frac{\sqrt{5}}{2}$

$\csc\theta = \frac{3\sqrt{5}}{5}$

$\cot\theta = \frac{2\sqrt{5}}{5}$

e. $\cot\theta = \frac{3}{8} \quad \theta \in III$

$\sin\theta = -\frac{8\sqrt{73}}{73}$

$\cos\theta = -\frac{3\sqrt{73}}{73}$

$\tan\theta = \frac{8}{3}$

$\sec\theta = -\frac{\sqrt{73}}{3}$

$\csc\theta = -\frac{\sqrt{73}}{8}$

f. $\csc\theta = \frac{4}{3} \quad \theta \in II$

$\sin\theta = \frac{3}{4}$

$\cos\theta = -\frac{\sqrt{65}}{9}$

$\tan\theta = -\frac{4\sqrt{65}}{65}$

$\sec\theta = -\frac{9\sqrt{65}}{65}$

$\cot\theta = -\frac{\sqrt{65}}{4}$

g. $\sin\theta = -\frac{12}{13} \quad \cos\theta > 0$

$\cos\theta = \frac{5}{13}$

$\tan\theta = -\frac{12}{5}$

$\sec\theta = \frac{13}{5}$

$\csc\theta = -\frac{13}{12}$

$\cot\theta = -\frac{12}{13}$

h. $\csc\theta = 3 \quad \tan\theta < 0$

$\sin\theta = \frac{1}{3}$

$\cos\theta = -\frac{2\sqrt{2}}{3}$

$\tan\theta = -\frac{\sqrt{2}}{4}$

$\sec\theta = -\frac{3\sqrt{2}}{4}$

$\cot\theta = -2\sqrt{2}$

i. $\cot\theta = -\frac{3}{4} \quad \sin\theta < 0$

$\sin\theta = -\frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = -\frac{4}{3}$

$\sec\theta = \frac{5}{3}$

$\csc\theta = -\frac{5}{4}$

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8. For the following coordinates on the terminal arm of an angle θ, find the exact value of all six trigonometric ratios

a. $A(4,7)$

$$\sin\theta = \frac{7\sqrt{65}}{65}$$
$$\cos\theta = \frac{4\sqrt{65}}{65}$$
$$\tan\theta = \frac{7}{4}$$
$$\csc\theta = \frac{\sqrt{65}}{7}$$
$$\sec\theta = \frac{\sqrt{65}}{4}$$
$$\cot\theta = \frac{4}{7}$$

b. $J(-8,-3)$

$$\sin\theta = -\frac{3\sqrt{73}}{73}$$
$$\cos\theta = -\frac{8\sqrt{73}}{73}$$
$$\tan\theta = \frac{3}{8}$$
$$\csc\theta = -\frac{\sqrt{73}}{3}$$
$$\sec\theta = -\frac{\sqrt{73}}{8}$$
$$\cot\theta = \frac{8}{3}$$

c. $C(5,-8)$

$$\sin\theta = -\frac{8\sqrt{89}}{89}$$
$$\cos\theta = \frac{5\sqrt{89}}{89}$$
$$\tan\theta = -\frac{8}{5}$$
$$\csc\theta = -\frac{\sqrt{89}}{8}$$
$$\sec\theta = \frac{\sqrt{89}}{5}$$
$$\cot\theta = -\frac{5}{8}$$

d. $D(-4,8)$

$$\sin\theta = \frac{2\sqrt{5}}{5}$$
$$\cos\theta = -\frac{\sqrt{5}}{5}$$
$$\tan\theta = -2$$
$$\csc\theta = \frac{\sqrt{5}}{2}$$
$$\sec\theta = -\sqrt{5}$$
$$\cot\theta = -\frac{1}{2}$$

e. $F(-2,4)$

$$\sin\theta = \frac{2\sqrt{5}}{5}$$
$$\cos\theta = -\frac{\sqrt{5}}{5}$$
$$\tan\theta = -2$$
$$\csc\theta = \frac{\sqrt{5}}{2}$$
$$\sec\theta = -\sqrt{5}$$
$$\cot\theta = -\frac{1}{2}$$

f. $G(-9,3)$

$$\sin\theta = \frac{\sqrt{10}}{10}$$
$$\cos\theta = -\frac{3\sqrt{10}}{10}$$
$$\tan\theta = -\frac{1}{3}$$
$$\csc\theta = \sqrt{10}$$
$$\sec\theta = -\frac{\sqrt{10}}{3}$$
$$\cot\theta = -3$$

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Learning Goal 4.2	Solving first- and second-degree equations over restricted domains and all real numbers.

1. Solve the following trigonometric equation on the specified domain for exact value(s) of θ, where possible. Otherwise round your answers to the nearest hundredth.

Proficient

a. $4\csc\theta - 5 = 3$, $0 \le \theta < 360$

$\theta_1 = 30°$
$\theta_2 = 150°$

b. $-3(5 - 4\sec\theta) = \sec\theta$, $0 \le \theta < 2\pi$

$\theta_1 \approx 0.75$
$\theta_2 \approx 5.54$

c. $\csc\theta + \frac{3}{4} = -\frac{2}{3}$, $0 \le \theta < 360$

$\theta_1 \approx 224.90°$
$\theta_2 \approx 315.10°$

d. $\sec\theta + 10 = 2 - 4\sec\theta$, $-\pi \le \theta < \pi$

$\theta_1 \approx 2.25$
$\theta_2 \approx -2.25$

e. $\sin\theta = \sqrt{3} - \sin\theta$, $0 \le \theta < 2\pi$

$\theta_1 = \frac{\pi}{3}$
$\theta_2 = \frac{2\pi}{3}$

f. $1 + \cos\theta = 1 - \cos\theta$, $-360 \le \theta < 0$

$\theta_1 = -90°$
$\theta_2 = -270°$

g. $\tan\theta = \sqrt{3} - 2\tan\theta$, $-\pi \le \theta < \pi$

$\theta_1 = \frac{\pi}{6}$
$\theta_2 = -\frac{5\pi}{6}$

h. $\sqrt{3}\sec\theta - 2 = 0$, $0 \le \theta < 2\pi$

$\theta_1 = \frac{\pi}{6}$
$\theta_2 = \frac{11\pi}{6}$

i. $7\cot\theta = \sqrt{3} + 5\cot\theta$, $-360 \le \theta < 0$

$\theta_1 \approx -130.89°$
$\theta_2 \approx -310.89°$

Extending

j. $2\sin^2\theta = 1$, $0 \le \theta < 2\pi$

$\theta_1 = \frac{\pi}{4}$ $\theta_2 = \frac{3\pi}{4}$
$\theta_3 = \frac{5\pi}{4}$ $\theta_4 = \frac{7\pi}{4}$

k. $\tan^2\theta - 4\tan\theta + 3 = 0$, $0 \le \theta < 2\pi$

$\theta_1 \approx 1.25$ $\theta_2 = \frac{\pi}{4}$
$\theta_3 \approx 4.39$ $\theta_4 = \frac{5\pi}{4}$

l. $\tan^2\theta - 3\tan\theta = 0$, $0 \le \theta < 2\pi$

$\theta_1 = 0$ $\theta_2 \approx 1.25$
$\theta_3 \approx 4.39$

m. $2\sin^2\theta + \sin\theta = 1$, $-2\pi \le \theta < 0$

$\theta_1 = -\frac{7\pi}{6}$ $\theta_2 = -\frac{\pi}{2}$
$\theta_3 = -\frac{11\pi}{6}$

n. $2\cos^2\theta + \cos\theta - 1 = 0$, $0 \le \theta < 2\pi$

$\theta_1 = \frac{\pi}{3}$ $\theta_2 = \pi$
$\theta_3 = \frac{5\pi}{3}$

o. $\csc^2\theta + 5\csc\theta + 6 = 0$, $-\pi \le \theta < \pi$

$\theta_1 = -\frac{\pi}{6}$ $\theta_2 \approx -0.34$
$\theta_3 = -\frac{5\pi}{6}$ $\theta_3 \approx -2.80$

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p. $2\cos^2\theta + 3\cos\theta + 1 = 0$ $0 \le \theta < 2\pi$ $\theta_1 = \frac{2\pi}{3}$ $\theta_2 = \pi$ $\theta_3 = \frac{4\pi}{3}$	q. $4\sin^2\theta - 1 = 0$ $0 \le \theta < 2\pi$ $\theta_1 = \frac{\pi}{6}$ $\theta_2 = \frac{7\pi}{6}$ $\theta_3 = \frac{5\pi}{6}$ $\theta_4 = \frac{11\pi}{6}$	r. $\sec^2\theta + 3\sec\theta = -2$ $0 \le \theta < 2\pi$ $\theta_1 = \pi$ $\theta_2 = \frac{2\pi}{3}$ $\theta_3 = \frac{4\pi}{3}$