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Chapter 6 Review

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

Proficient

1. Prove the identity.

a. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

$\tan^2 x - \sin^2 x$	$\tan^2 x \sin^2 x$
$= \dfrac{\sin^2 x}{\cos^2 x} - \sin^2 x$ $= \dfrac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$ $= \dfrac{\sin^2 x\,(1 - \cos^2 x)}{\cos^2 x}$ $= \tan^2 x\,(1 - \cos^2 x)$ $= \tan^2 x \sin^2 x$	

b. $\dfrac{\sec x - \cos x}{\tan x} = \sin x$

$\dfrac{\sec x - \cos x}{\tan x}$	$\sin x$
$= \dfrac{1/\cos x - \cos x}{\sin x/\cos x}$ $= \dfrac{(1 - \cos^2 x)/\cos x}{\sin x/\cos x}$ $= \dfrac{1 - \cos^2 x}{\cos x} \times \dfrac{\cos x}{\sin x}$ $= (1 - \cos^2 x) \times \dfrac{1}{\sin x}$ $= \dfrac{\sin^2 x}{\sin x}$ $= \sin x$	

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c. $\dfrac{\cos x + \sin x \tan x}{\sin x \sec x} = \csc x$

$\dfrac{\cos x + \sin x \tan x}{\sin x \sec x}$	$\csc x$
$= \dfrac{\cos x + \sin x \tan x}{\sin x \times \left({}^{1}/_{\cos x}\right)}$	$= \dfrac{1}{\sin x}$
$= \dfrac{\cos x + \left(\sin x \times {}^{\sin x}/_{\cos x}\right)}{{}^{\sin x}/_{\cos x}}$	
$= \dfrac{\cos x + \left({}^{\sin^2 x}/_{\cos x}\right)}{{}^{\sin x}/_{\cos x}}$	
$= \dfrac{{}^{(\cos^2 x + \sin^2 x)}/_{\cos x}}{{}^{\sin x}/_{\cos x}}$	
$= \dfrac{\cos^2 x + \sin^2 x}{\cos x} \times \dfrac{\cos x}{\sin x}$	
$= \dfrac{1}{\cos x} \times \dfrac{\cos x}{\sin x}$	
$= 1 \times \dfrac{1}{\sin x}$	
$= \dfrac{1}{\sin x}$	

d. $\dfrac{\csc x}{\tan x + \cot x} = \cos x$

$\dfrac{\csc x}{\tan x + \cot x}$	$\cos x$
$= \dfrac{{}^{1}/_{\sin x}}{\sin x / \cos x + {}^{\cos x}/_{\sin x}}$	
$= \dfrac{{}^{1}/_{\sin x}}{{}^{(\sin^2 x + \cos^2 x)}/_{\sin x \cos x}}$	
$= \dfrac{{}^{1}/_{\sin x}}{{}^{1}/_{\sin x \cos x}}$	
$= \dfrac{1}{\sin x} \times \dfrac{\sin x \cos x}{1}$	
$= \cos x$	

e. $\sin x + \cos x \cot x = \csc x$

$\sin x + \cos x \cot x$	$\csc x$
$= \sin x + \cos x \times \dfrac{\cos x}{\sin x}$	$= \dfrac{1}{\sin x}$
$= \sin x + \dfrac{\cos^2 x}{\sin x}$	
$= \dfrac{\sin^2 x + \cos^2 x}{\sin x}$	
$= \dfrac{1}{\sin x}$	

f. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$

$\csc^2 x + \sec^2 x$	$\csc^2 x \sec^2 x$
$= \dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}$	$= \dfrac{1}{\sin^2 x \cos^2 x}$
$= \dfrac{\cos^2 x}{\sin^2 x \cos^2 x} + \dfrac{\sin^2 x}{\sin^2 x \cos^2 x}$	
$= \dfrac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$	
$= \dfrac{1}{\sin^2 x \cos^2 x}$	

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Extending

g. $\dfrac{\cot x}{\csc x - 1} = \dfrac{\csc x + 1}{\cot x}$

$\dfrac{\cot x}{\csc x - 1}$	$\dfrac{\csc x + 1}{\cot x}$
$= \dfrac{\cot x}{\csc x - 1} \times \dfrac{\csc x + 1}{\csc x + 1}$ $= \dfrac{\cot x\,(\csc x + 1)}{\csc^2 x - 1}$ $= \dfrac{\cot x\,(\csc x + 1)}{\cot^2 x}$ $= \dfrac{\csc x + 1}{\cot x}$	

h. $\dfrac{\sin 2x}{2 - 2\cos^2 x} = \cot x$

$\dfrac{\sin 2x}{2 - 2\cos^2 x}$	$\cot x$
$= \dfrac{\sin 2x}{2(1 - \cos^2 x)}$ $= \dfrac{2\sin x \cos x}{2(1 - \cos^2 x)}$ $= \dfrac{2\sin x \cos x}{2\sin^2 x}$ $= \dfrac{\sin x \cos x}{\sin^2 x}$ $= \dfrac{\cos x}{\sin x}$	$= \dfrac{\cos x}{\sin x}$

i. $\dfrac{1 - \cos x}{\sin^2 x} = \dfrac{1}{1 + \cos x}$

$\dfrac{1 - \cos x}{\sin^2 x}$	$\dfrac{1}{1 + \cos x}$
	$= \dfrac{1}{1 + \cos x} \times \dfrac{1 - \cos x}{1 - \cos x}$ $= \dfrac{1 - \cos x}{1 - \cos^2 x}$ $= \dfrac{1 - \cos x}{\sin^2 x}$

j. $\dfrac{\sin 2x}{\cos x} + \dfrac{\cos 2x}{\sin x} = \csc x$

$\dfrac{\sin 2x}{\cos x} + \dfrac{\cos 2x}{\sin x}$	$\csc x$
$= \dfrac{2\sin x \cos x}{\cos x} + \dfrac{\cos 2x}{\sin x}$ $= \dfrac{2\sin x}{1} + \dfrac{\cos 2x}{\sin x}$ $= \dfrac{2\sin^2 x}{\sin x} + \dfrac{\cos 2x}{\sin x}$ $= \dfrac{2\sin^2 x + 1 - 2\sin^2 x}{\sin x}$ $= \dfrac{1}{\sin x}$	$= \dfrac{1}{\sin x}$

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k. $\dfrac{1}{\sec x + \tan x} = \dfrac{1 - \sin x}{\cos x}$

$\dfrac{1}{\sec x + \tan x}$	$\dfrac{1 - \sin x}{\cos x}$
$= \dfrac{1}{1/\cos x + \sin x/\cos x}$ $= \dfrac{1}{(1 + \sin x)/\cos x}$ $= \dfrac{\cos x}{1 + \sin x} \times \dfrac{1 - \sin x}{1 - \sin x}$ $= \dfrac{\cos x\,(1 - \sin x)}{1 - \sin^2 x}$ $= \dfrac{1 - \sin x}{\cos x}$	

l. $\dfrac{\sin x \cos x}{1 + \cos x} = \dfrac{1 - \cos x}{\tan x}$

$\dfrac{\sin x \cos x}{1 + \cos x}$	$\dfrac{1 - \cos x}{\tan x}$
$= \dfrac{\sin x \cos x}{1 + \cos x} \times \dfrac{1 - \cos x}{1 - \cos x}$ $= \dfrac{\sin x \cos x\,(1 - \cos x)}{1 - \cos^2 x}$ $= \dfrac{\sin x \cos x\,(1 - \cos x)}{\sin^2 x}$ $= \dfrac{\cos x\,(1 - \cos x)}{\sin x}$	$= \dfrac{1 - \cos x}{\sin x/\cos x}$ $= \dfrac{1 - \cos x/1}{\sin x/\cos x}$ $= \dfrac{1 - \cos x}{1} \times \dfrac{\cos x}{\sin x}$ $= \dfrac{\cos x\,(1 - \cos x)}{\sin x}$

m. $\dfrac{\cos 2x}{\sin x} = \dfrac{\cot^2 x - 1}{\csc x}$

$\dfrac{\cos 2x}{\sin x}$	$\dfrac{\cot^2 x - 1}{\csc x}$
$= \dfrac{\cos^2 x - \sin^2 x}{\sin x}$	$= \dfrac{\cos^2 x/\sin^2 x - 1}{\csc x}$ $= \dfrac{(\cos^2 x - \sin^2 x)/\sin^2 x}{\csc x}$ $= \dfrac{(\cos^2 x - \sin^2 x)/\sin^2 x}{1/\sin x}$ $= \dfrac{\cos^2 x - \sin^2 x}{\sin^2 x}$ $= \dfrac{\cos^2 x - \sin^2 x}{\sin x}$

n. $\dfrac{\cot x - 1}{1 - \tan x} = \dfrac{\csc x}{\sec x}$

$\dfrac{\cot x - 1}{1 - \tan x}$	$\dfrac{\csc x}{\sec x}$
$= \dfrac{\cos x/\sin x - 1}{1 - \sin x/\cos x}$ $= \dfrac{(\cos x - \sin x)/\sin x}{(\cos x - \sin x)/\cos x}$ $= \dfrac{\cos x - \sin x}{\sin x} \times \dfrac{\cos x}{\cos x - \sin x}$ $= \dfrac{\cos x}{\sin x}$	$= \dfrac{1/\sin x}{1/\cos x}$ $= \dfrac{1}{\sin x} \times \dfrac{\cos x}{1}$ $= \dfrac{\cos x}{\sin x}$

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o. $(1-\sin x)(\sec x+\tan x)=\dfrac{1}{\sec x}$

$(1-\sin x)(\sec x+\tan x)$	$\dfrac{1}{\sec x}$
$=\sec x+\tan x-\sin x\sec x$ $-\sin x\tan x$	$=\dfrac{1}{1/\cos x}$
$=\sec x+\tan x-\sin x\left(\dfrac{1}{\cos x}\right)$ $-\sin x\left(\dfrac{\sin x}{\cos x}\right)$	$=\dfrac{1}{1}\times\dfrac{\cos x}{1}$
$=\sec x+\tan x-\dfrac{\sin x}{\cos x}-\dfrac{\sin^2 x}{\cos x}$	$=\cos x$
$=\sec x+\tan x-\tan x-\dfrac{\sin^2 x}{\cos x}$	
$=\dfrac{1}{\cos x}-\dfrac{\sin^2 x}{\cos x}$	
$=\dfrac{1-\sin^2 x}{\cos x}$	
$=\dfrac{\cos^2 x}{\cos x}$	
$=\cos x$	

p. $\sin 2x\,(\tan x+\cot x)=2$

$\sin 2x\,(\tan x+\cot x)$	2
$=2\sin x\cos x\,(\tan x+\cot x)$	
$=2\sin x\cos x\left(\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}\right)$	
$=2\sin x\cos x\left(\dfrac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right)$	
$=2\sin x\cos x\left(\dfrac{1}{\sin x\cos x}\right)$	
$=2$	

q. $\dfrac{\cot x}{\sin x\csc x}=-\sec x$

$\dfrac{\cot x}{\sin x-\csc x}$	$-\sec x$
$=\dfrac{\left(\cos x/\sin x\right)}{\sin x-\left(1/\sin x\right)}$	$=\dfrac{1}{-\cos x}$
$=\dfrac{\left(\cos x/\sin x\right)}{\left(\sin^2 x-1\right)/\sin x}$	
$=\dfrac{\cos x}{\sin x}\times\dfrac{\sin x}{\sin^2 x-1}$	
$=\dfrac{\cos x}{\sin x}\times\dfrac{\sin x}{-\cos^2 x}$	
$=\dfrac{\cos x}{1}\times\dfrac{1}{-\cos^2 x}$	
$=\dfrac{1}{-\cos x}$	

r. $\dfrac{\sin 2x}{1+\cos 2x}=\dfrac{\sec^2 x-1}{\tan x}$

$\dfrac{\sin 2x}{1+\cos 2x}$	$\dfrac{\sec^2 x-1}{\tan x}$
$=\dfrac{2\sin x\cos x}{1+\cos 2x}$	$=\dfrac{\tan^2 x}{\tan x}$
$=\dfrac{2\sin x\cos x}{1+(2\cos^2 x-1)}$	$=\tan x$
$=\dfrac{2\sin x\cos x}{2\cos^2 x}$	$=\dfrac{\sin x}{\cos x}$
$=\dfrac{\sin x\cos x}{\cos^2 x}$	
$=\dfrac{\sin x}{\cos x}$	

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s. $\dfrac{2\cos x + 2\cos^2 x}{\sin 2x} = \dfrac{\sin x}{1-\cos x}$

$\dfrac{2\cos x + 2\cos^2 x}{\sin 2x}$	$\dfrac{\sin x}{1-\cos x}$
$= \dfrac{2\cos x\,(1+\cos x)}{\sin 2x}$	$= \dfrac{\sin x}{1-\cos x} \times \dfrac{1+\cos x}{1+\cos x}$
$= \dfrac{2\cos x\,(1+\cos x)}{2\sin x\cos x}$	$= \dfrac{\sin x\,(1+\cos x)}{1-\cos^2 x}$
$= \dfrac{1+\cos x}{\sin x}$	$= \dfrac{\sin x\,(1+\cos x)}{\sin^2 x}$
	$= \dfrac{1+\cos x}{\sin x}$

t. $\dfrac{1}{1+\sin x} = \sec^2 x - \dfrac{\tan x}{\cos x}$

$\dfrac{1}{1+\sin x}$	$\sec^2 x - \dfrac{\tan x}{\cos x}$
$= \dfrac{1}{1+\sin x} \times \dfrac{1-\sin x}{1-\sin x}$	$= \dfrac{1}{\cos^2 x} - \dfrac{\tan x}{\cos x}$
$= \dfrac{1-\sin x}{1-\sin^2 x}$	$= \dfrac{1}{\cos^2 x} - \dfrac{\left(\sin x/\cos x\right)}{\cos x}$
	$= \dfrac{1}{\cos^2 x} - \dfrac{\left(\sin x/\cos x\right)}{\left(\cos x/1\right)}$
	$= \dfrac{1}{\cos^2 x} - \left(\dfrac{\sin x}{\cos x} \times \dfrac{1}{\cos x}\right)$
	$= \dfrac{1}{\cos^2 x} - \dfrac{\sin x}{\cos^2 x}$
	$= \dfrac{1-\sin x}{\cos^2 x}$
	$= \dfrac{1-\sin x}{1-\sin^2 x}$

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u. $\dfrac{\sin x + \tan x}{1 + \cos x} = \dfrac{\sin 2x}{2\cos^2 x}$

$\dfrac{\sin x + \tan x}{1 + \cos x}$	$\dfrac{\sin 2x}{2\cos^2 x}$
$= \dfrac{\sin x + \tan x}{1 + \cos x} \times \dfrac{1 - \cos x}{1 - \cos x}$ $= \dfrac{(\sin x + \tan x)(1 - \cos x)}{1 - \cos^2 x}$ $= \dfrac{(\sin x + \tan x)(1 - \cos x)}{\sin^2 x}$ $= \dfrac{\sin x - \sin x \cos x + \tan x - \tan x \cos x}{\sin^2 x}$ $= \dfrac{\sin x - \sin x \cos x + \tan x - \sin x}{\sin^2 x}$ $= \dfrac{-\sin x \cos x + \tan x}{\sin^2 x}$ $= \dfrac{(-\sin x \cos^2 x + \sin x)/\cos x}{\sin^2 x/1}$ $= \dfrac{-\sin x \cos^2 x + \sin x}{\cos x} \times \dfrac{1}{\sin^2 x}$ $= \dfrac{\sin x(1 - \cos^2 x)}{\cos x} \times \dfrac{1}{\sin^2 x}$ $= \dfrac{1 - \cos^2 x}{\cos x} \times \dfrac{1}{\sin x}$ $= \dfrac{\sin^2 x}{\cos x} \times \dfrac{1}{\sin x}$ $= \dfrac{\sin x}{\cos x}$	$= \dfrac{2\sin x \cos x}{2\cos^2 x}$ $= \dfrac{\sin x}{\cos x}$