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## Chapter 7 and 8 Algebra Review

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

## Learning Goal 8.1

Solving exponential and logarithmic equations with the same base and different bases, including base $e$.

1. Write each expression as a single logarithm. Show all work and evaluate where possible.

| Developing |  |
| :---: | :---: |
| a. $\log (x-3)+\log (x+4)$ | b. $\log _{4} 8+\log _{4} 32$ |
| c. $\log _{2} 96-\log _{2} 3$ | d. $\log 25+2 \log 4+\log 5-\log 2$ |
| Proficient |  |
| e. $\log _{2} x^{3}-4 \log _{2} x-\log _{2} \sqrt{x}$ | f. $4 \log _{6} y^{2}+\log _{6} y-\frac{2}{3} \log _{6} y$ |
| g. $\log _{6}(216 \times \sqrt[4]{36})$ | h. $4 \log _{6} y^{2}+\log _{6} y-\frac{2}{3} \log _{6} y$ |
| Extending |  |
| i. $\quad \log _{6} 2 x^{7}+\log _{6} 3 x^{2}+\log _{6}\left(\frac{9}{x^{5}}\right)$ | j. $\log _{2} 5 x^{2} y^{3}-\log _{2} 20 x^{4} y+\log _{2} 2 x y^{6}$ |
| k. $\log _{4}\left(x^{2} y\right)^{2}+5 \log _{4} x^{3} y^{4}+\log _{4}\left(\frac{1}{x^{3} y^{2}}\right)$ | I. $6 \log _{3} x y-\log _{3} x y^{2}-\log _{3} \sqrt[3]{x^{4} y}$ |
| $\text { m. } \frac{1}{2} \log 4 x \sqrt{y}-\log 25 x^{2} \sqrt{y}$ | n. $\quad \log _{7} x^{4}+\frac{1}{3}\left(\log _{7} x^{2}-\log _{7} \sqrt{5 x}\right)$ |
| o. $\frac{\log 16 x^{8}}{4}-\frac{\log 27 x}{3}$ | p. $\frac{\log _{9} x^{4} y^{8}}{2}+\frac{\log _{9} x^{12} y^{15}}{3}$ |

2. Expand each logarithm as far as possible, including simplifying all powers to the smallest possible base.

## Developing

a. $\log _{5} \sqrt{x y^{3}}$
b. $\quad \log _{7}\left(x^{4} \sqrt{y^{3}}\right)$
c. $\log _{12}\left(x y^{2} z^{5}\right)^{3}$
d. $\quad \log _{8}\left(\frac{x^{3}}{\sqrt{y z^{5}}}\right)$
e. $\log _{7}\left(49 \sqrt[3]{x^{5}}\right)$
f. $\quad \log _{5}\left(\frac{\sqrt[3]{y^{7}}}{125 x}\right)$

## Proficient

g. $\log _{4}\left(\frac{x^{3} y}{4 z}\right)$
i. $\ln \left(\frac{\sqrt[3]{24}}{\sqrt{50}}\right)$
h. $\log \left(\frac{100 \sqrt[3]{x^{4}}}{y^{2}}\right)$
j. $\quad \log _{2}\left(\frac{3 x^{6}}{96 y^{2}}\right)$
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3. Write each expression in terms of $a$ when $a=\log _{5} 12$.

## Developing

a. $\log _{5} 12^{7}$
b. $\log _{5} 60$
c. $\log _{5} 144$
d. $\log _{5} 12 / 5$
e. $\log _{5} 1 / 12$
f. $\log _{5} \sqrt{12}$
4. Solve showing all steps. State an exact answer then estimate where applicable. State any restrictions on the domain and check for extraneous roots.

| Developing |  |
| :---: | :---: |
| a. $\quad 3^{x+5}=27$ | b. $3^{2 x-1}=9$ |
| c. $9^{x+5}=27^{-2 x}$ | d. $16^{2 x-5}=32$ |
| e. $\left(\frac{1}{3}\right)^{x}=27^{x-1}$ | f. $\sqrt{8}=64^{x}$ |
| g. $\quad \frac{1}{49}=7^{x-1}$ | h. $\sqrt{8}=64^{x}$ |
| i. $\quad 2^{x}=9$ | j. $5 \times 3^{x}=135$ |
| Proficient |  |
| k. $\quad 2^{x}=3^{x-1}$ | l. $6^{x}=10^{x}$ |
| m. $5^{x}=2\left(3^{x}\right)$ | ก. $5^{x}=7^{x-2}$ |
| o. $64^{4 x}=16^{x+5}$ | p. $9^{x-7}=27^{2 x-9}$ |
| q. $125^{6 x+2}=25^{8 x+1}$ | r. $8^{x+2}=\left(\frac{1}{4}\right)^{x+3}$ |
| s. $12^{3 x}=1000$ | t. $\quad 7^{x+2}=441$ |
| Extending |  |
| u. $3\left(5^{x}\right)=6^{x-1}$ | v. $2\left(6^{x}\right)=5^{x+1}$ |
| w. $3^{2 x}=7^{x+1}$ | x. $2\left(6^{x}\right)=5^{x+1}$ |
| y. $3^{2 x / 3}=350$ | z. $2\left(6^{x+2}\right)=3^{2 x-3}$ |

5. Solve showing all steps. State an exact answer then estimate where applicable. State any restrictions on the domain and check for extraneous roots.

## Developing

a. $\log _{4}(5 x+1)=\log _{4}(x+17)$
b. $\log _{4} x=5$
c. $\log _{5} x+6=2$
d. $2 \log _{2} x=10$
e. $\log _{6}(x+3)+2=5$
f. $3 \log _{5} x=\log _{5} 125$

## Proficient

g. $2 \log _{2}(x-5)=6$
i. $3 \log _{6} x=\log _{6} 9+\log _{6} 24$
k. $\log _{2} x^{2}-\log _{2} 5=\log _{2} 20$
m. $5 \log _{3} x-\log _{3} x=8$
h. $3 \log _{5} x=\log _{5} 125$
j. $3 \log _{5} x=\log _{5} 125$
l. $\log _{4} x+2 \log _{4} x=6$
n. $\log _{3}(4 x+9)=5$
$\qquad$

## Chapter 7 and 8 Algebra Review

| Extending |  |
| :--- | ---: |
| o. $\log _{2}(x+1)+\log _{2} x=\log _{2} 5$ | p. $\log (x+5)+\log x=\log 2$ |
| q. $\log (x+3)+\log (x-5)=1$ | r. $\log (x-4)+\log x=\log 0.1$ |
| s. $\log x+\log (x+1)=\log 3$ | t. $\log x+\log (x+3)=\log 8$ |
| u. $\log (5 x)-\log (x-1)=1$ | v. $\log _{8}(6 x+2)+\log _{8}(x-3)=2$ |
| w. $\log _{6}(x-3)+\log _{6}(x+6)=2$ | x. $\log _{2}(4 x+10)-\log _{2} x=3$ |
| y. $\log (2 x+6)=1+\log (x-1)$ | z. $\log _{4}(x-4)+\log _{4}(x+2)=2$ |

## Extending

6. In 1990, the population of a town was 32000 and was increasing at a rate of $3.5 \%$ per year.
a. Write an equation to represent the population of this town, $P$, as a function of the number of years, $n$, since 1990.
b. What is the population of the town in 2022?
c. How long until the population of the town reaches 1000000 ?
7. A geometric sequence is a sequence in which each term is found by multiplying the preceding term by the same value, a common ratio. The sequence is

$$
2,-0.8,0.32,-0.128
$$

a. Determine the common ratio.
b. Find the $10^{\text {th }}$ term
c. Write a formula for the $n^{\text {th }}$ term.
d. What number of term (approximately) would -0.000000008796 be?
8. A thermocouple is used to measure extremely high temperatures. When a thermocouple is placed on the element of an electric range, the resulting temperature $T$, in degrees Celcius, can be modelled by $T=150 \log 4 x$ where $x$ is the time in seconds.
a. Estimate the time when the temperature reaches $230^{\circ} \mathrm{C}$ without a calculator.
b. Calculate the exact time when the temperature reaches $275^{\circ} \mathrm{C}$.
9. The amount, $A$ dollars, in a bank account is represented by the equation $A=1080 \times 1.0045^{t / 12}$. Assuming the account is using compound interest,
a. What was the principle amount?
b. What is the compounding period?
c. What is the annual interest rate?
d. Exactly how long until the return on investment is quadrupled?

