Name: ______________________________________ Date: ________________

Chapter 7 and 8 Algebra Review

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 8.1	Solving exponential and logarithmic equations with the same base and different bases, including base e.

1. Write each expression as a single logarithm. Show all work and evaluate where possible.

Developing	
a. $\log(x-3)+\log(x+4)$ $=\log(x^2+x-12)$	b. $\log_4 8+\log_4 32$ $=4$
c. $\log_2 96-\log_2 3$ $=5$	d. $\log 25+2\log 4+\log 5-\log 2$ $=3$
Proficient	
e. $\log_2 x^3-4\log_2 x-\log_2 \sqrt{x}$ $=\log_2\left(\frac{1}{x^{3/2}}\right)$	f. $4\log_6 y^2+\log_6 y-\frac{2}{3}\log_6 y$ $=\log_6\left(y^{25/3}\right)$
g. $\log_6(216\times\sqrt[4]{36})$ $=\frac{7}{2}$	h. $4\log_6 y^2+\log_6 y-\frac{2}{3}\log_6 y$ $=\log_6\left(y^{25/3}\right)$
Extending	
i. $\log_6 2x^7+\log_6 3x^2+\log_6\left(\frac{9}{x^5}\right)$ $=\log_6 54x^4$	j. $\log_2 5x^2y^3-\log_2 20x^4y+\log_2 2xy^6$ $=\log_2\left(\frac{y^8}{2x}\right)$
k. $\log_4(x^2y)^2+5\log_4 x^3y^4+\log_4\left(\frac{1}{x^3y^2}\right)$ $=\log_4 x^{16}y^{20}$	l. $6\log_3 xy-\log_3 xy^2-\log_3\sqrt[3]{x^4y}$ $=\log_3\left(\sqrt[3]{(xy)^{11}}\right)$
m. $\frac{1}{2}\log 4x\sqrt{y}-\log 25x^2\sqrt{y}$ $=\log\left(\frac{2}{25x\times\sqrt[4]{x^2y}}\right)$	n. $\log_7 x^4+\frac{1}{3}(\log_7 x^2-\log_7\sqrt{5x})$ $=\log_7\left(\frac{x^{9/2}}{5^{1/6}}\right)$
o. $\frac{\log 16x^8}{4}-\frac{\log 27x}{3}$ $=\log\left(\frac{2x^{5/3}}{3}\right)$	p. $\frac{\log_9 x^4y^8}{2}+\frac{\log_9 x^{12}y^{15}}{3}$ $=\log_9(x^6y^9)$

Name: ___________________________________ Date: _______________

Chapter 7 and 8 Algebra Review

2. Expand each logarithm as far as possible, including simplifying all powers to the smallest possible base.

Developing	
a. $\log_5 \sqrt{xy^3}$ $= \frac{1}{2}\log_5 x + \frac{3}{2}\log_5 y$	b. $\log_7 \left(x^4\sqrt{y^3}\right)$ $= 4\log_7 x + \frac{3}{2}\log_7 y$
c. $\log_{12}(xy^2z^5)^3$ $= 3\log_{12} x + 6\log_{12} y + 15\log_{12} z$	d. $\log_8\left(\frac{x^3}{\sqrt{yz^5}}\right)$ $= 3\log_8 x - \frac{1}{2}\log_8 y - \frac{5}{2}\log_8 z$
e. $\log_7\left(49\sqrt[3]{x^5}\right)$ $= 2 + \frac{5}{3}\log_7 x$	f. $\log_5\left(\frac{\sqrt[3]{y^7}}{125x}\right)$ $= \frac{7}{3}\log_5 y - \log_5 x - 3$
Proficient	
g. $\log_4\left(\frac{x^3y}{4z}\right)$ $= 3\log_4 x + \log_4 y - \log_4 z - 1$	h. $\log\left(\frac{100\sqrt[3]{x^4}}{y^2}\right)$ $= 2 + \frac{4}{3}\log x - 2\log y$
i. $\ln\left(\frac{\sqrt[3]{24}}{\sqrt{50}}\right)$ $= \ln 2 + \frac{1}{3}\ln 3 - \ln 5 - \frac{1}{2}\ln 2$	j. $\log_2\left(\frac{3x^6}{96y^2}\right)$ $= 6\log_2 x - 2\log_2 y - 5$

3. Write each expression in terms of a when $a = \log_5 12$.

Developing		
a. $\log_5 12^7$ $= 7a$	b. $\log_5 60$ $= 1 + a$	c. $\log_5 144$ $= 2a$
d. $\log_5 {}^{12}/_5$ $= a - 1$	e. $\log_5 {}^{1}/_{12}$ $= -a$	f. $\log_5 \sqrt{12}$ $= \frac{a}{2}$

4. Solve showing all steps. State an exact answer then estimate where applicable. State any restrictions on the domain and check for extraneous roots.

Developing	
a. $3^{x+5} = 27$ $x \in \mathbb{R}$ $\quad$ $x = -2$	b. $3^{2x-1} = 9$ $x \in \mathbb{R}$ $\quad$ $x = \frac{3}{2}$
c. $9^{x+5} = 27^{-2x}$ $x \in \mathbb{R}$ $\quad$ $x = -\frac{5}{4}$	d. $16^{2x-5} = 32$ $x \in \mathbb{R}$ $\quad$ $x = \frac{25}{8}$

Name: ______________________________________ Date: ________________

Chapter 7 and 8 Algebra Review

e. $\left(\frac{1}{3}\right)^{x} = 27^{x-1}$ $x \in \mathbb{R}$ $x = \frac{3}{4}$	f. $\sqrt{8} = 64^{x}$ $x \in \mathbb{R}$ $x = \frac{1}{4}$
g. $\frac{1}{49} = 7^{x-1}$ $x \in \mathbb{R}$ $x = -1$	h. $\sqrt{8} = 64^{x}$ $x \in \mathbb{R}$ $x = \frac{1}{4}$
i. $2^{x} = 9$ $x \in \mathbb{R}$ $x = \log_2 9$	j. $5 \times 3^{x} = 135$ $x \in \mathbb{R}$ $x = 3$
Proficient	
k. $2^{x} = 3^{x-1}$ $x \in \mathbb{R}$ $x = \frac{\log 3}{\log\left(3/2\right)}$	l. $6^{x} = 10^{x}$ $x \in \mathbb{R}$ $x = 0$
m. $5^{x} = 2(3^{x})$ $x \in \mathbb{R}$ $x = \frac{\log 2}{\log\left(5/3\right)}$	n. $5^{x} = 7^{x-2}$ $x \in \mathbb{R}$ $x = \frac{\log 49}{\log\left(7/5\right)}$
o. $64^{4x} = 16^{x+5}$ $x \in \mathbb{R}$ $x = 1$	p. $9^{x-7} = 27^{2x-9}$ $x \in \mathbb{R}$ $x = \frac{13}{4}$
q. $125^{6x+2} = 25^{8x+1}$ $x \in \mathbb{R}$ $x = -2$	r. $8^{x+2} = \left(\frac{1}{4}\right)^{x+3}$ $x \in \mathbb{R}$ $x = \frac{\log 49}{\log\left(7/5\right)}$
s. $12^{3x} = 1000$ $x \in \mathbb{R}$ $x = \log_{1728} 1000$	t. $7^{x+2} = 441$ $x \in \mathbb{R}$ $x = \log_7 441 - 2$
Extending	
u. $3(5^{x}) = 6^{x-1}$ $x \in \mathbb{R}$ $x = \frac{\log 18}{\log\left(6/5\right)}$	v. $2(6^{x}) = 5^{x+1}$ $x \in \mathbb{R}$ $x = \frac{\log\left(5/2\right)}{\log\left(6/5\right)}$
w. $3^{2x} = 7^{x+1}$ $x \in \mathbb{R}$ $x = \frac{\log 7}{\log\left(9/7\right)}$	x. $2(6^{x}) = 5^{x+1}$ $x \in \mathbb{R}$ $x = \frac{\log\left(5/2\right)}{\log\left(6/5\right)}$
y. $3^{2x/3} = 350$ $x \in \mathbb{R}$ $x = \log_7\left(1750\sqrt{14}\right)$	z. $2(6^{x+2}) = 3^{2x-3=}$ $x \in \mathbb{R}$ $x = \log_{3/2}\left(\frac{8}{3}\right)$

Name: ___________________________________ Date: _______________

Chapter 7 and 8 Algebra Review

5. Solve showing all steps. State an exact answer then estimate where applicable. State any restrictions on the domain and check for extraneous roots.

Developing	
a. $\log_4(5x+1) = \log_4(x+17)$ $x > -\frac{1}{5}$ $x = 4$	b. $\log_4 x = 5$ $x > 0$ $x = 1024$
c. $\log_5 x + 6 = 2$ $x > 0$ $x = \frac{1}{625}$	d. $2\log_2 x = 10$ $x > 0$ $x = 32$
e. $\log_6(x+3) + 2 = 5$ $x > -3$ $x = 213$	f. $3\log_5 x = \log_5 125$ $x > 0$ $x = 5$
Proficient	
g. $2\log_2(x-5) = 6$ $x > 5$ $x = 13$	h. $3\log_5 x = \log_5 125$ $x > 0$ $x = 5$
i. $3\log_6 x = \log_6 9 + \log_6 24$ $x > 0$ $x = 6$	j. $3\log_5 x = \log_5 125$ $x > 0$ $x = 5$
k. $\log_2 x^2 - \log_2 5 = \log_2 20$ $x > 0$ $x = 10$	l. $\log_4 x + 2\log_4 x = 6$ $x > 0$ $x = 16$
m. $5\log_3 x - \log_3 x = 8$ $x > 0$ $x = 9$	n. $\log_3(4x+9) = 5$ $x > -\frac{9}{4}$ $x = \frac{117}{2}$
Extending	
o. $\log_2(x+1) + \log_2 x = \log_2 5$ $x > 0$ $x = \frac{-1+\sqrt{21}}{2}$	p. $\log(x+5) + \log x = \log 2$ $x > 0$ $x = \frac{-5+\sqrt{33}}{2}$
q. $\log(x+3) + \log(x-5) = 1$ $x > 5$ $1+\sqrt{26}$	r. $\log(x-4) + \log x = \log 0.1$ $x > 4$ $x = \frac{20+\sqrt{410}}{10}$
s. $\log x + \log(x+1) = \log 3$ $x > 0$ $x = \frac{-1+\sqrt{13}}{2}$	t. $\log x + \log(x+3) = \log 8$ $x > 0$ $x = \frac{-3+\sqrt{41}}{2}$
u. $\log(5x) - \log(x-1) = 1$ $x > 1$ $x = 2$	v. $\log_8(6x+2) + \log_8(x-3) = 2$ $x > 3$ $x = 5$
w. $\log_6(x-3) + \log_6(x+6) = 2$ $x > 3$ $x = 6$	x. $\log_2(4x+10) - \log_2 x = 3$ $x > 0$ $x = \frac{5}{2}$
y. $\log(2x+6) = 1 + \log(x-1)$ $x > 1$ $x = 2$	z. $\log_4(x-4) + \log_4(x+2) = 2$ $x > 4$ $x = 6$

Name: ___________________________________ Date: _______________

Chapter 7 and 8 Algebra Review

Extending
6. In 1990, the population of a town was 32 000 and was increasing at a rate of 3.5% per year. a. Write an equation to represent the population of this town, P, as a function of the number of years, n, since 1990. b. What is the population of the town in 2022? c. How long until the population of the town reaches 1 000 000? a. $P(n) = 32\,000\,(1.035)^n$ b. $P(32) = 96\,214$ c. $n \approx 100$ years
7. A geometric sequence is a sequence in which each term is found by multiplying the preceding term by the same value, a common ratio. The sequence is $2, -0.8, 0.32, -0.128.$ a. Determine the common ratio. b. Find the 10^{th} term c. Write a formula for the n^{th} term. d. What number of term (approximately) would -0.000000008796 be? a. $-{}^{2}/_{5}$ b. -0.0000524288 c. $t_n = 2 \times \left(-{}^{2}/_{5}\right)^{n-1}$ d. t_{22}
8. A thermocouple is used to measure extremely high temperatures. When a thermocouple is placed on the element of an electric range, the resulting temperature T, in degrees Celcius, can be modelled by $T = 150 \log 4x$ where x is the time in seconds. a. Estimate the time when the temperature reaches 230℃ without a calculator. b. Calculate the exact time when the temperature reaches 275℃. a. $230 = \log(4x)^{150}$ $10^{230} = (4x)^{150}$ $225 = 150 \times \frac{3}{2}$ $\left(10^{3/2}\right)^{150} \approx (4x)^{150}$ $10^{3/2} \approx 4x$ $x \approx \frac{10 \times 3}{4}$ $x \approx 7.5$ s b. $230 = \log(4x)^{150}$ $10^{230} = (4x)^{150}$ $4x = \sqrt[150]{10^{230}}$ $x = \frac{\sqrt[150]{10^{230}}}{4}$ $x \approx 8.5$ s
9. The amount, A dollars, in a bank account is represented by the equation $A = 1080 \times 1.0045^{t/12}$. Assuming the account is using compound interest, a. What was the principle amount? b. What is the compounding period? c. What is the annual interest rate? d. Exactly how long until the return on investment is quadrupled? a. \$1080 b. Monthly c. 5.4% d. ≈ 309 years