

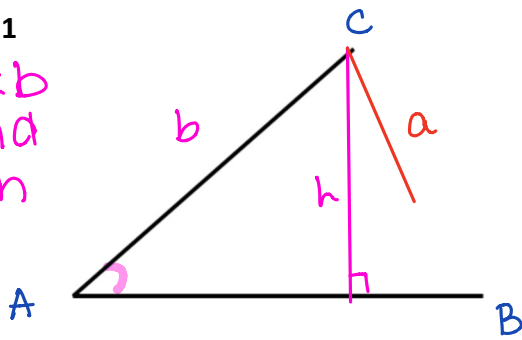
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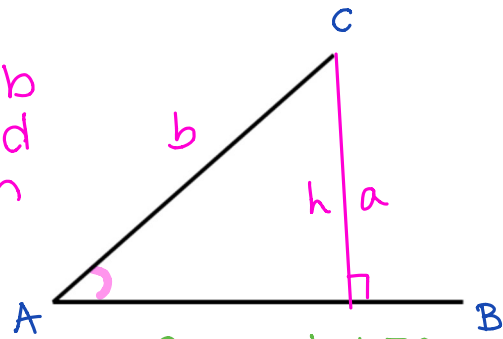
Learning Goal 2.3

Use of sine and cosine laws to solve non-right triangles, including ambiguous cases.

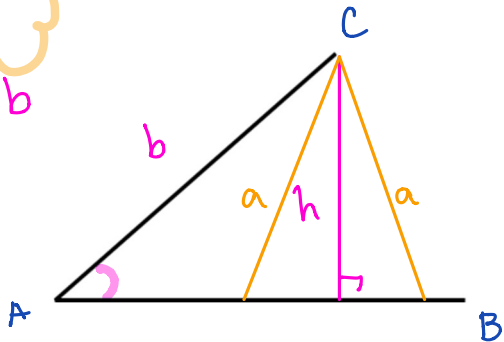
The Ambiguous Case when we're given 2 sides that don't contain an angle.

Case 1
 $a < b$
and
 $a < h$


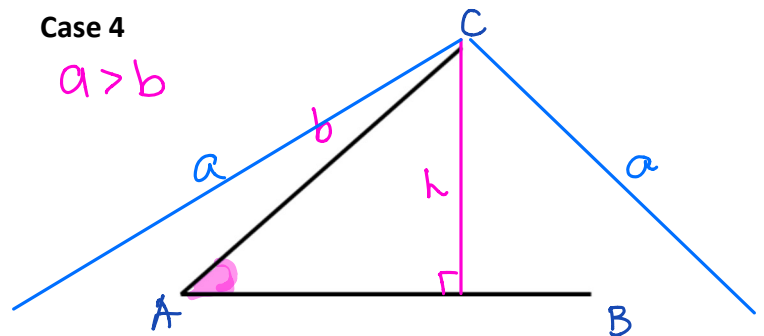
NO Solutions.
(no triangle.)

Case 2
 $a < b$
and
 $a = h$


One solution
(unique triangle)

Case 3
 $h < a < b$


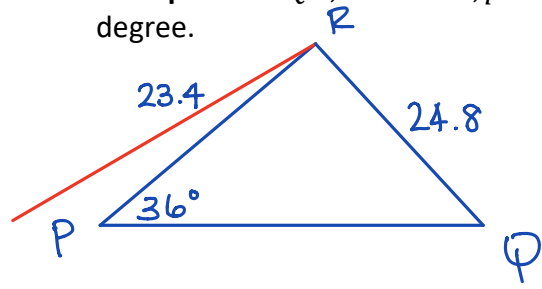
2 possible
triangles.
- you need to find the
answer for both triangles.

Case 4
 $a > b$


two triangles
but you'll know which
one you're dealing
with

$\angle A$ is either acute
or obtuse.

Example In $\triangle PQR$, $\angle P = 36^\circ$, $p = 24.8$ m and $q = 23.4$ m. Determine the measure of $\angle R$ to the nearest degree.



$p > q$ but $\angle P$ is acute so only one triangle.

$$\frac{\sin P}{p} = \frac{\sin Q}{q}$$

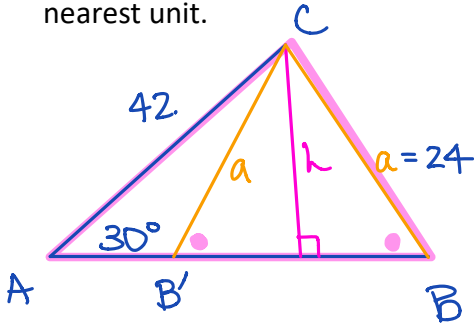
$$23.4 \times \frac{\sin 36}{24.8} = \frac{\sin Q}{23.4} \times 23.4$$

$$\frac{23.4 \times \sin 36}{24.8} = \sin Q$$

$$Q = \sin^{-1} \left(\frac{23.4 \times \sin 36}{24.8} \right) \approx 34^\circ$$

$$\begin{aligned} P + Q + R &= 180^\circ \\ 36^\circ + 34^\circ + R &= 180^\circ \\ R &= 180^\circ - 36^\circ - 34^\circ \\ &= 110^\circ \end{aligned}$$

Example In $\triangle ABC$, $\angle A = 30^\circ$, $a = 24$ cm and $b = 42$ m. Solve the triangle, expressing all answers to the nearest unit.



$a < b$ Find $\angle B$.

$$\sin A = \frac{h}{b}$$

$$42 \times \sin 30 = \frac{h}{42} \times 42$$

$$h = 42 \times \sin 30 = 21$$

$h < a < b \Rightarrow 2$ triangles (case 3)

$$\begin{aligned} B' &= 180 - B \\ &= 180 - 61 \\ &= 119^\circ \end{aligned}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$42 \times \frac{\sin 30}{24} = \frac{\sin B}{42} \times 42$$

$$\sin B = \frac{42 \times \sin 30}{24}$$

$$B = \sin^{-1} \left(\frac{42 \times \sin 30}{24} \right)$$

$$= 61^\circ$$