Name: ___________________________ Date: ____________________

Learning Goal 2.2	Limits at infinity and the definition of the derivative

We will apply these methods to **four** different types of limits:

1. Basic
2. one-sided $\lim_{x\to a^-}$ $\lim_{x\to a^+}$
3. Infinite $\lim_{x\to a} f(x) = \infty$
4. Limits at infinity.

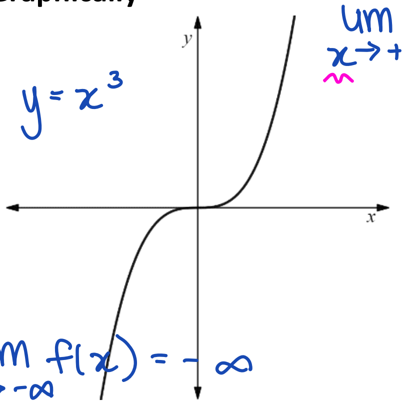
$\lim_{x\to\infty} f(x)$ - very Big Positive values.

$\lim_{x\to-\infty} f(x)$ - very Big Negative values.

Graphically

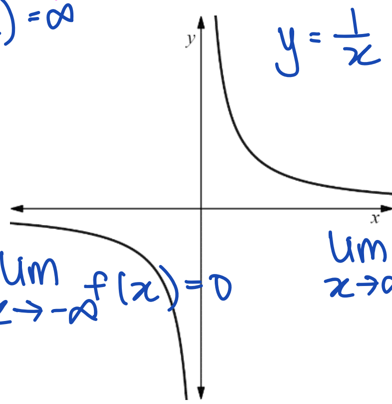
$y = x^3$

$\lim_{x\to+\infty} f(x) = \infty$



$\lim_{x\to-\infty} f(x) = -\infty$

$y = \frac{1}{x}$



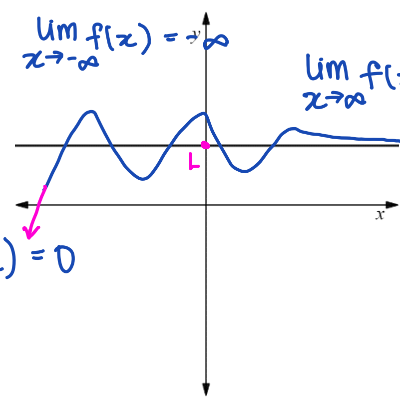
$\lim_{x\to-\infty} f(x) = 0$

$\lim_{x\to\infty} f(x) = 0$

Horizontal asymptote @ $y = 0$

$\lim_{x\to-\infty} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = L$



Horizontal asymptote at $y = L$

Fact #1

can be a fraction

If r is a positive rational number and number and c is any real number, then

$$\lim_{x\to\infty} \frac{c}{x^r} = 0 \quad \text{and} \quad \lim_{x\to-\infty} \frac{c}{x^r} = 0$$

a. $\lim_{x\to\infty} 2x^4 - \cancel{x^2} - \cancel{8x} = \infty$

$= \lim_{x\to\infty} 2x^4 - \lim_{x\to\infty} x^2 - \lim_{x\to\infty} 8x$

$= \cancel{\infty - \infty} - \infty$

$\lim_{x\to\infty} x(2x^3 - \cancel{x} - \cancel{8})$

b. $\lim_{x\to-\infty} \frac{1}{3}x^5 + \cancel{2x^3} - \cancel{x^2} + \cancel{8}$

$= \lim_{x\to-\infty} \frac{1}{3}x^5$

$= -\infty$

If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial of degree n ($a_n \neq 0$), then

$$\lim_{x\to\pm\infty} p(x) = a_n x^n$$

Definition

The function $f(x)$ will have a horizontal asymptote at $y = L$ if either of the following are true.

$\lim_{x\to\infty} f(x) = L$ OR $\lim_{x\to-\infty} f(x) = L$

a. $\lim_{x\to\infty} \frac{2x^4 - \cancel{x^2} - \cancel{8x}}{-5x^4 + \cancel{7}}$

$= \lim_{x\to\infty} \frac{2\cancel{x^4}}{-5\cancel{x^4}}$

$= -\frac{2}{5}$

$= \lim_{x\to\infty} \frac{\frac{2x^4}{x^4} - \frac{x^2}{x^4} - \frac{8x}{x^4}}{-\frac{5x^4}{x^4} + \frac{7}{x^4}}$

$= -\frac{2}{5}$

Horizontal asymptote @ $-\frac{2}{5}$

b. $\lim_{x\to-\infty} \frac{2x^4 - \cancel{x^2} - \cancel{8x}}{-5x^4 + \cancel{7}}$

$= \lim_{x\to-\infty} \frac{2x^4}{-5x^4}$

$= -\frac{2}{5}$

asymptote runs in both directions.

c. $\lim_{x\to\infty} \frac{\cancel{4x^2} + x^6}{\cancel{1} - 5x^3}$

$= \lim_{x\to\infty} \frac{x^6}{-5x^3}$

$= \lim_{x\to\infty} \frac{x^3}{-5}$

$= -\infty$

No asymptote.

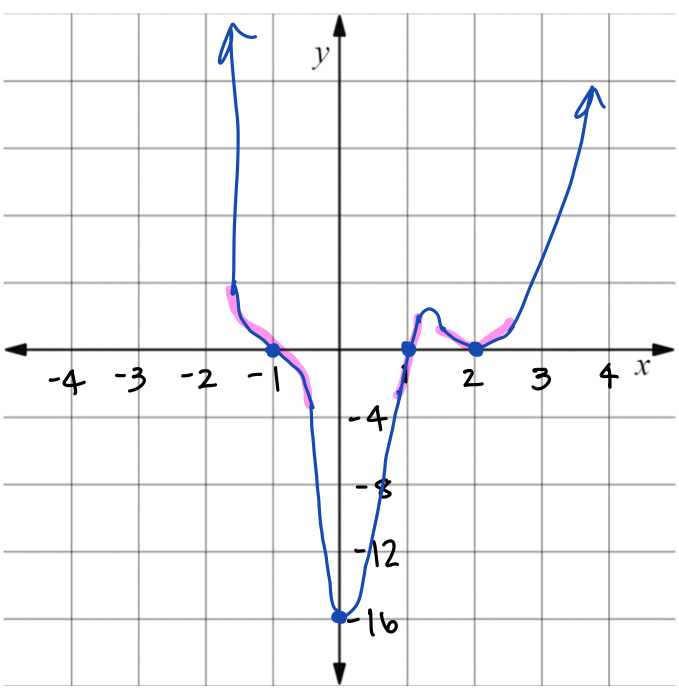
d. $\lim_{x\to-\infty} \frac{\cancel{4x^2} + x^6}{\cancel{1} - 5x^3}$

$= \lim_{x\to-\infty} \frac{x^3}{-5}$

$= \infty$

degree 8

Example Sketch the graph of $y = (x-2)^4(x+1)^3(x-1)$ by finding its intercepts and its limits as $x \to \pm\infty$.



1. x-intercept(s) $y = 0$

$(x-2)^4(x+1)^3(x-1) = 0$

$x = 2$ mult. 4 $x = -1$ mult 3 $x = 1$ mult 1

2. y-intercept $x = 0$

$(-2)^4(1)^3(-1) = -16$

3. end behaviour (limits @ ∞)

$\lim_{x\to\infty} (x-\cancel{2})^4(x+\cancel{1})^3(x-\cancel{1})$

$= \lim_{x\to\infty} x^4 \times x^3 \times x$

$= \lim_{x\to\infty} x^8 = \infty$

$\lim_{x\to-\infty} x^8 = \infty$

Assignment # 1 – 9, 13 – 21, 31, 41 – 47 Quiz Next Day!