

Name: _____

Date: _____

Learning Goal 3.3

Convert standard form of the quadratic equation to vertex form by completing the square.

Standard Form

→

Vertex Form

$$ax^2 + bx + c$$

$$a(x-p)^2 + q$$

Results in just one extra step:

Example: $f(x) = 5x^2 + 30x + 41$

0. Factor.

$$= 5(x^2 + 6x) + 41$$

1. Group and Complete the Square:

$$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$$

2. Add in the rest of the equation:

$$= 5(x^2 + 6x + 9 - 9) + 41$$

3. Isolate the Perfect Square Trinomial:

$$= 5(x^2 + 6x + 9) - 45 + 41$$

4. Simplify

$$f(x) = 5(x+3)^2 - 4$$

Example Convert the following quadratic equations to vertex form.

a. $f(x) = 3x^2 - 12x - 9$

- $= 3(x^2 - 4x) - 9$
- $\left(-\frac{4}{2}\right)^2 = (-2)^2 = 4$
- $= 3(x^2 - 4x + 4 - 4) - 9$
- $= 3(x^2 - 4x + 4) - 12 - 9$
- $f(x) = 3(x - 2)^2 - 21$

b. $g(x) = -5x^2 - 70x + 24$

- $= -5(x^2 + 14x) + 24$
- $\left(\frac{14}{2}\right)^2 = (7)^2 = 49$
- $= -5(x^2 + 14x + 49 - 49) + 24$
- $= -5(x^2 + 14x + 49) + 245 + 24$
- $= -5(x + 7)^2 + 269$

c. $h(x) = 2x^2 - 20x + 0$

- $= 2(x^2 - 10x)$
- $\left(-\frac{10}{2}\right)^2 = (-5)^2 = 25$
- $= 2(x^2 - 10x + 25 - 25)$
- $= 2(x^2 - 10x + 25) - 50$
- $= 2(x - 5)^2 - 50$

d. $y = -3x^2 - 8x - 24$

- $= -3\left(x^2 + \frac{8}{3}x\right) - 24$
- $\left(\frac{8/3}{2}\right)^2 = \left(\frac{8}{6}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$
 $\frac{8}{3} \times \frac{1}{2}$
- $= -3\left(x^2 + \frac{8}{3}x + \frac{16}{9} - \frac{16}{9}\right) - 24$
- $= -3\left(x^2 + \frac{8}{3}x + \frac{16}{9}\right) + \frac{16}{3} - 24 - \frac{72}{3}$
- $y = -3\left(x + \frac{4}{3}\right)^2 - \frac{56}{3}$