

$a \neq 1$

Name: _____

Date: _____

Learning Goal 3.3

Convert standard form of the quadratic equation to vertex form by completing the square.

More Questions – Solutions

Convert the following quadratic equations to vertex form. Find the domain, range, vertex, y – intercept, the equation for the axis of symmetry, and the maximum or minimum value.

a. $f(x) = 2x^2 - 10x - 38$

b. $g(x) = 5x^2 + 18x - 68$

$f(x) = 2(x^2 - 5x) - 38$

$g(x) = 5\left(x^2 + \frac{18}{5}x\right) - 68$

 $x^2 - 5x + \underline{\hspace{2cm}}$ is a perfect square trinomial?

$x^2 - 5x + \frac{25}{4}$

$x^2 + \frac{18}{5}x + \underline{\hspace{2cm}}$

is a perfect square trinomial?

$$\begin{aligned} f(x) &= 2\left(x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right) - 38 \\ &= 2\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{2} - 38 \\ &= 2\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{2} - \frac{76}{2} \\ &= 2\left(x^2 - 5x + \frac{25}{4}\right) - \frac{101}{2} \\ &= 2\left(x - \frac{5}{2}\right)^2 - \frac{101}{2} \end{aligned}$$

$$\begin{aligned} g(x) &= 5\left(x^2 + \frac{18}{5}x + \frac{81}{25} - \frac{81}{25}\right) - 68 \\ &= 5\left(x^2 + 4x + \frac{81}{25}\right) - \frac{81}{5} - \frac{340}{5} \\ &= 5\left(x^2 + 4x + \frac{81}{25}\right) - \frac{421}{5} \\ &= 5\left(x + \frac{9}{5}\right)^2 - \frac{421}{5} \end{aligned}$$

Domain	$\{x x \in \mathbb{R}\}$
Range	$\{y y \geq -\frac{101}{2}, y \in \mathbb{R}\}$
Vertex	$\left(\frac{5}{2}, -\frac{101}{2}\right)$
y – intercept	$y = -38$
Axis of Symmetry	$x = \frac{5}{2}$
Max or Min? Value?	Minimum $y = -\frac{101}{2}$

Domain	$\{x x \in \mathbb{R}\}$
Range	$\{y y \geq -\frac{421}{25}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{9}{5}, -\frac{421}{5}\right)$
y – intercept	$y = -68$
Axis of Symmetry	$x = -\frac{9}{5}$
Max or Min? Value?	Minimum $y = -\frac{421}{5}$

$a \neq 1$

a. $h(x) = -2x^2 - 14x - 66$

$$h(x) = -2(x^2 + 7x) - 66$$

$$x^2 + 7x + \underline{\hspace{2cm}}$$

is a perfect square trinomial?

$$x^2 + 7x + \frac{49}{4}$$

$$\begin{aligned} h(x) &= -2\left(x^2 + 8x + \frac{49}{4} - \frac{49}{4}\right) - 66 \\ &= -2\left(x^2 + 8x + \frac{49}{4}\right) + \frac{49}{2} - \frac{132}{2} \\ &= -2\left(x^2 + 8x + \frac{49}{4}\right) - \frac{83}{2} \\ &= -2\left(x + \frac{7}{2}\right)^2 - \frac{83}{2} \end{aligned}$$

Domain	$\{x x \in \mathbb{R}\}$
Range	$\{y y \leq -\frac{83}{2}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{7}{2}, -\frac{83}{2}\right)$
y – intercept	$y = -66$
Axis of Symmetry	$x = -\frac{7}{2}$
Max or Min? Value?	Maximum $y = -\frac{83}{2}$

b. $j(x) = -10x^2 + 21x - 80$

$$j(x) = -10\left(x^2 - \frac{21}{10}x\right) - 80$$

$$x^2 - \frac{21}{10}x + \underline{\hspace{2cm}}$$

is a perfect square trinomial?

$$x^2 - \frac{21}{10}x + \frac{441}{400}$$

$$\begin{aligned} j(x) &= -10\left(x^2 - 2x + \frac{441}{400} - \frac{441}{400}\right) - 80 \\ &= -10\left(x^2 - 2x + \frac{441}{400}\right) + \frac{441}{40} - \frac{3200}{40} \\ &= -10\left(x^2 - 2x + \frac{441}{400}\right) - \frac{2759}{40} \\ &= -10\left(x - \frac{21}{20}\right)^2 - \frac{2759}{40} \end{aligned}$$

Domain	$\{x x \in \mathbb{R}\}$
Range	$\{y y \leq -\frac{2759}{40}, y \in \mathbb{R}\}$
Vertex	$\left(\frac{21}{20}, -\frac{2759}{40}\right)$
y – intercept	$y = -80$
Axis of Symmetry	$x = \frac{21}{20}$
Max or Min? Value?	Maximum $y = -\frac{2759}{40}$