Name: ___________________________ Date: ____________________

Learning Goal 3.2	Applying derivatives to trigonometric and exponential functions.

The Number 'e' - irrational. 2.71828...

↳ the base for natural growth & decay

$$e^x \Leftrightarrow \log_e x = \ln x$$

$$\log_a b = c \Leftrightarrow a^c = b$$

Some Handy Derivatives

$y = e^x$	$y = a^x$	$y = \ln x$
$\frac{dy}{dx} = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$ $= \lim_{h\to 0} \frac{e^x e^h - e^x}{h}$ $= \lim_{h\to 0} \frac{e^x(e^h - 1)}{h}$ $= e^x \underbrace{\lim_{h\to 0} \frac{e^h - 1}{h} = \frac{0}{0} \text{ indeterminate.}}_{\text{Numerically } = 1}$ $= e^x$ so $\boxed{\frac{d}{dx}(e^x) = e^x}$	$a^x = e^n$ ← need this! $\ln(a^x) = \ln(e^n)$ $x \ln a = n \ln e$ $\log_e e = 1$ $\ln e = 1$ $n = x \ln a$. $\frac{d}{dx}(a^x) = \frac{d}{dx}(e^{x\ln a})$ $= e^{x \ln a} \times \left(\frac{d}{dx} x \ln a\right)$ $= e^{x \ln a} \times \ln a$. $\boxed{= a^x \times \ln a}$.	$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$ $y = \log_a x$ $\boxed{\frac{dy}{dx} = \frac{1}{x} \times \frac{1}{\ln a}}$

Just like before, these are now rules that do not need to be derived every time. They can be used as fact!

Assignment # 5, 15, 21, 23, 36, 46, 47, 61 Quiz Next Day!

Example Differentiate.

a. $y = \dfrac{e^x}{1+x}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(1+x) = 1$

$$\frac{dy}{dx} = \frac{(1+x)e^x - e^x(1)}{(1+x)^2}$$

$$= \frac{e^x(1+x-1)}{(1+x)^2}$$

$$= \frac{xe^x}{(1+x)^2}$$

b. $y = e^{c\tan\sqrt{x}}$

$$\frac{dy}{dx} = e^{c\tan\sqrt{x}} \times \left[\frac{d}{dx} c\tan\sqrt{x}\right]$$

$$= e^{c\tan\sqrt{x}} \times c\left[\frac{d}{dx}\tan\sqrt{x}\right]$$

$$= e^{c\tan\sqrt{x}} \times c\sec^2\sqrt{x} \times \left[\frac{d}{dx}\sqrt{x}\right]$$

$$= e^{c\tan\sqrt{x}} \times c\sec^2\sqrt{x} \times \frac{1}{2\sqrt{x}} \times \left[\frac{d}{dx}x\right]$$

c. $f(x) = \cos(e^{\pi x})$

$$f'(x) = -\sin(e^{\pi x}) \times \left[\frac{d}{dx}e^{\pi x}\right]$$

$$= -\sin(e^{\pi x}) \times e^{\pi x} \times \left[\frac{d}{dx}\pi x\right]$$

$$= -\sin(e^{\pi x}) \times e^{\pi x} \times \pi$$

d. $g(x) = \sqrt{1 + xe^{-2x}}$

$$= (1+xe^{-2x})^{\frac{1}{2}}$$

$$g'(x) = \frac{1}{2}(1+xe^{-2x})^{-1/2} \times \left[\frac{d}{dx}(\cancel{1}+xe^{-2x})\right]$$

$$= \frac{1}{2}(1+xe^{-2x})^{-1/2} \times \left[\underbrace{x(e^{-2x})'}_{e^{-2x} \times \left[\frac{d}{dx}-2x\right] = -2e^{-2x}} + e^{-2x}(\cancel{x})'\right]$$

$$= \frac{1}{2}(1+xe^{-2x})^{-1/2} \times \left[-2xe^{-2x} + e^{-2x}\right]$$

$$= \frac{e^{-2x}}{2}(1+xe^{-2x})^{-1/2}(1-2x)$$