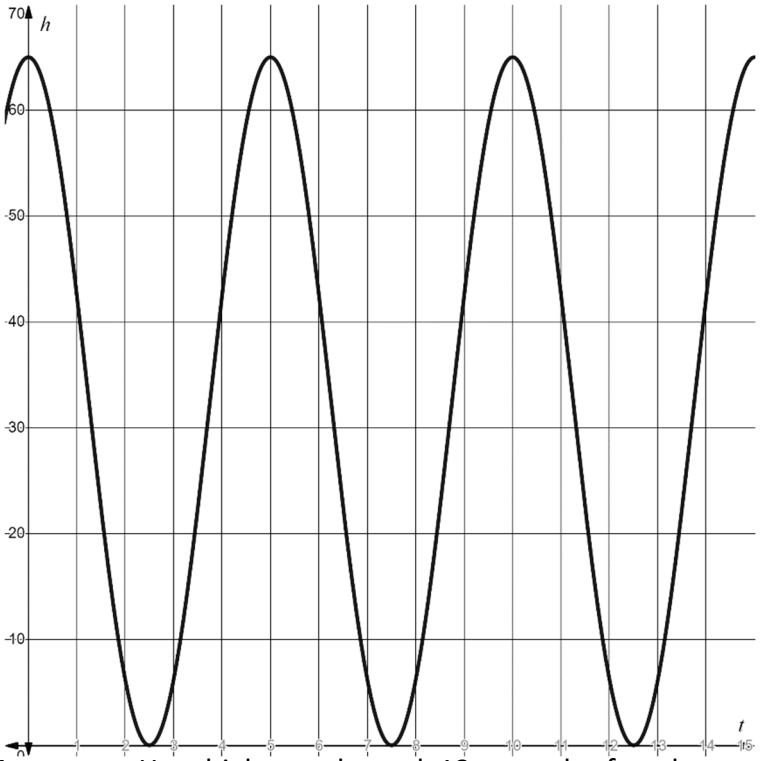
Name: _____ Date: _____

Learning Goal 5.1	Graphing primary trigonometric functions, including transformations and characteristics.

More Questions - Solutions

1. A tack is stuck in the top of a wheel with a diameter of 65 cm. When the wheel is moving slowly it rotates 12 times per minute.
   a. Sketch a graph of showing the path of the tack for 15 sec.



   b. Write an equation to model the path of the tack.

$$a = \frac{65}{2} \qquad \frac{2\pi}{b} = 5 \qquad b = \frac{2\pi}{5} \qquad c = \frac{65}{2}$$

$$h(t) = \frac{65}{2}\cos\frac{2\pi}{5}t + \frac{65}{2}$$

   c. State the domain and range of the function.

$$\{x|x \geq 0, x \in \mathbb{R}\}$$
$$\{y|0 \leq y \leq 65, y \in \mathbb{R}\}$$

   d. At what time did the tack reach a height of 24 cm in its first revolution? (round to nearest hundredth of a second.)

$$\frac{65}{2}\cos\frac{2\pi}{5}t + \frac{65}{2} = 24$$
$$\frac{65}{2}\cos\frac{2\pi}{5}t = -\frac{17}{2}$$
$$\cos\frac{2\pi}{5}t = -\frac{17}{65}$$
$$\frac{2\pi}{5}t = \cos^{-1}\left(-\frac{17}{65}\right)$$
$$t = \frac{5}{2\pi}\cos^{-1}\left(-\frac{17}{65}\right)$$
$$t \approx 1.46 \text{ seconds}$$

   e. How high was the tack 12 seconds after the wheel began to move?

$$h = \frac{65}{2}\cos\frac{2\pi}{5}(12) + \frac{65}{2}$$
$$h = \frac{65}{2}\cos\frac{24\pi}{5} + \frac{65}{2}$$
$$h \approx 6.2 \text{ cm}$$

2. The depth of the water in a harbor can be modeled by the function

$$h(t) = 3\sin\frac{\pi(t-2)}{6.4} + 7,$$

where h is the ocean depth in meters and t is the time of day.

a. How deep is the water at $8{:}17$ pm?

$$8{:}17 \text{ pm} = 20\frac{17}{60} = \frac{1217}{60}$$

$$h = 3\sin\frac{\pi\left(\frac{1217}{60} - 2\right)}{6.4} + 7$$

$$h = 3\sin\frac{\pi\left(\frac{1097}{60}\right)}{6.4} + 7$$

$$h = 3\sin\frac{1097\pi}{384} + 7$$

$$h \approx 8.3 \text{ m}$$

b. At what time(s) does the depth reach 9 m in a 24 hour period?

$$3\sin\frac{\pi(t-2)}{6.4} + 7 = 9$$

$$3\sin\frac{\pi(t-2)}{6.4} = 2$$

$$\sin\frac{\pi(t-2)}{6.4} = \frac{2}{3}$$

$$\frac{\pi(t-2)}{6.4} = \sin^{-1}\left(\frac{2}{3}\right)$$

$$t - 2 = \frac{6.4}{\pi}\sin^{-1}\left(\frac{2}{3}\right)$$

$$t = \frac{6.4}{\pi}\sin^{-1}\left(\frac{2}{3}\right) + 2$$

$$t_1 \approx \frac{6.4}{\pi}(0.7297) + 2$$

$$t_1 \approx 3.5 \text{ hrs}$$

$$\approx 3{:}30 \text{ am}$$

$$t_2 \approx \frac{6.4}{\pi}(\pi - 0.7297) + 2$$

$$t_2 \approx 6.9 \text{ hrs}$$

$$\approx 6{:}54 \text{ am}$$

$$t_3 \approx \frac{6.4}{\pi}(2\pi + 0.7297) + 2$$

$$t_3 \approx 16.3 \text{ hrs}$$

$$\approx 4{:}18 \text{ pm}$$

$$t_4 \approx \frac{6.4}{\pi}(3\pi - 0.7297) + 2$$

$$t_4 \approx 19.7 \text{ hrs}$$

$$\approx 7{:}42 \text{ pm}$$