

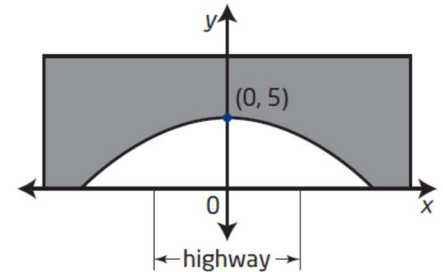
Name: _____

Date: _____

Learning Goal 9.2

Solving quadratic inequalities.

1. A highway goes under a bridge formed by a parabolic arch, as shown. The highest point of the arch is 5 m high. The road is 10 m wide, and the minimum height of the bridge over the road is 4 m. Determine the quadratic function that models the parabolic arch of the bridge.



If the minimum height is 4 m, then the points $(-5, 4)$ and $(5, 4)$ must both lie on the curve.

Find vertex form of the equation:

$$y = a(x - p)^2 + q$$

$$y = a(x - 0)^2 + 5$$

$$y = ax^2 + 5$$

Then find the expansion/compression factor by using one of the points you found:

$$4 = a(5)^2 + 5$$

$$4 = 25a + 5$$

$$-1 = 25a$$

$$a = -\frac{1}{25}$$

$$y = -\frac{1}{25}x^2 + 5$$

Now the bridge goes above the highway, so I want the point $(0, 0)$ to be false:

$$0 \nabla -\frac{1}{25}(0)^2 + 5$$

$$0 \nabla 0 + 5$$

$$0 \geq 5$$

$$y \geq -\frac{1}{25}x^2 + 5$$

$$\{y | y > 0, y \in \mathbb{R}\}$$

2. To raise money, the student council sells candy – grams each year. From past experience, they expect to sell 400 candy – grams at a price of \$4 each. They have also learned from experience that each \$0.50 increase in the price causes a drop in sales of 20 candy – grams. Write an equation that models this situation. Suppose the student council needs revenue of at least \$1 800. Solve an inequality to find all the possible prices that will achieve the fundraising goal.

Let R be the revenue in dollars, and x be the number of \$0.50 price changes

$$\text{Revenue} = \text{cost} \times \text{price}$$

$$R = (4 + 0.5x)(400 - 20x)$$

$$1\,800 < (4 + 0.5x)(400 - 20x)$$

$$1\,800 < 1600 - 80x + 200x - 10x^2$$

$$1\,800 < 1600 + 120x - 10x^2$$

$$0 < -200 + 120x - 10x^2$$

$$0 < -10x^2 + 120x - 200$$

$$0 < -10(x^2 - 12x + 20)$$

$$0 < -10(x - 2)(x - 10)$$

So one factor needs to be positive and the other negative.

$$\begin{array}{ll} x - 2 > 0 & x - 10 < 0 \\ x > 2 & x < 10 \end{array}$$

$$2 < x < 10$$

$$\begin{array}{ll} x - 2 < 0 & x - 10 > 0 \\ x < 2 & x > 10 \end{array}$$

No solutions

$$\{x \mid 2 < x < 10, x \in \mathbb{R}\}$$

The cost needs to be raised somewhere in between \$1.00 and \$5.00 for them to meet their revenue goals.