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Chapter 3 Review

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 3.1	Given a quadratic function, identify the transformation that the graph has undergone from the standard graph of $y = x^2$

1. Identify the transformations of each graph, then graph it without using a table of values.

Developing	
a. $a(x) = x^2 - 4$ down 4 https://www.desmos.com/calculator/hqm0iodkp9	b. $b(x) = x^2 + 5$ up 5 https://www.desmos.com/calculator/wzqcukguy9
c. $c(x) = x^2 - 1$ down 1 https://www.desmos.com/calculator/tojoncidqy	d. $d(x) = x^2 + 2$ up 2 https://www.desmos.com/calculator/aipykygius
e. $y = x^2 - 3$ down 3 https://www.desmos.com/calculator/41clv61kcu	f. $f(x) = x^2 + 6$ up 6 https://www.desmos.com/calculator/ycwgsitw27
g. $g(x) = (x - 1)^2$ right 1 https://www.desmos.com/calculator/htlvgilq6k	h. $h(x) = (x + 4)^2$ left 4 https://www.desmos.com/calculator/tsufsh10pd
i. $y = (x - 5)^2$ right 5 https://www.desmos.com/calculator/wvnerxhkzx	j. $j(x) = (x + 2)^2$ left 2 https://www.desmos.com/calculator/38wl67ddl0
k. $k(x) = (x - 3)^2$ right 3 https://www.desmos.com/calculator/utmleqvw5t	l. $y = (x + 1)^2$ left 1 https://www.desmos.com/calculator/i8ppmqgn7a
Proficient	
a. $a(x) = (x - 5)^2 + 4$ right 5, up 4 https://www.desmos.com/calculator/9nrvsbrl62	b. $b(x) = (x - 3)^2 + 2$ right 3, up 2 https://www.desmos.com/calculator/wvts7vg3dq
c. $c(x) = (x - 2)^2 + 3$ right 2, up 3 https://www.desmos.com/calculator/wvts7vg3dq	d. $d(x) = (x - 1)^2 - 3$ right 1, down 3 https://www.desmos.com/calculator/pc8vlrrbwd
e. $y = (x - 4)^2 - 1$ right 4, down 1 https://www.desmos.com/calculator/fwtaolomh1	f. $f(x) = (x - 3)^2 - 5$ right 3, down 5 https://www.desmos.com/calculator/qjbv2hmaia

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g. $g(x) = (x+3)^2 - 5$ left 3, down 5 https://www.desmos.com/calculator/ubc4qcpwcp	h. $h(x) = (x+5)^2 - 2$ left 5, down 2 https://www.desmos.com/calculator/adawansedy
i. $y = (x+1)^2 - 6$ left 1, down 6 https://www.desmos.com/calculator/lgzt2rqd9r	j. $j(x) = (x+2)^2 + 1$ left 2, up 1 https://www.desmos.com/calculator/su2lj0rvny
k. $k(x) = (x+6)^2 + 3$ left 6, up 3 https://www.desmos.com/calculator/z39jyingdd	l. $y = (x+4)^2 + 2$ left 4, up 2 https://www.desmos.com/calculator/z39jyingdd
m. $m(x) = -(x-1)^2 + 3$ mirrored over $x-$ axis, right 1, up 3 https://www.desmos.com/calculator/gmzjddk6ay	n. $n(x) = -(x-7)^2 - 2$ mirrored over $x-$ axis, right 7, down 2 https://www.desmos.com/calculator/yj4i3uuwcx
o. $y = -(x-2)^2 + 4$ mirrored over $x-$ axis, right 2, up 4 https://www.desmos.com/calculator/ecd47ictvu	p. $p(x) = -(x+3)^2 - 2$ mirrored over $x-$ axis, left 3, down 2 https://www.desmos.com/calculator/5lqm3lxp8p
q. $q(x) = -(x+5)^2 + 1$ mirrored over $x-$ axis, left 5, up 1 https://www.desmos.com/calculator/s6m3yawas1	r. $r(x) = -(x+2)^2 - 5$ mirrored over $x-$ axis, left 2, down 5 https://www.desmos.com/calculator/inry7vcvru
Extending	
a. $a(x) = 2(x+3)^2 - 8$ vertical compression/horizontal expansion by a factor of 2, left 3, down 8 https://www.desmos.com/calculator/nobkxz8s5l	b. $b(x) = -4(x+1)^2 + 4$ mirrored over the $x-$ axis, vertical compression/horizontal expansion by a factor of 4, left 1, up 4 https://www.desmos.com/calculator/jwhhkmocjn
c. $c(x) = \frac{1}{2}(x-2)^2 - 3$ vertical expansion/horizontal compression by a factor of 2, right 2, down 3 https://www.desmos.com/calculator/qok3hyd3je	d. $d(x) = -2(x-1)^2 - 5$ mirrored over the $x-$ axis, vertical compression/horizontal expansion by a factor of 2, right 1, down 5 https://www.desmos.com/calculator/lgnmo2qpax
e. $y(x) = 3(x-5)^2 + 1$ vertical compression/horizontal expansion by a factor of 3, right 5, up 1 https://www.desmos.com/calculator/igq1atqlab	f. $f(x) = -0.2(x+2)^2 - 1.5$ mirrored over the $x-$ axis, vertical expansion/horizontal compression by a factor of 5, left 2, down 1.5 https://www.desmos.com/calculator/qg6yb4xyre

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g. $g(x) = \frac{2}{3}(x+1)^2 + 6$ vertical expansion/horizontal compression by a factor of $^3/_2$, left 1, up 6 https://www.desmos.com/calculator/2ivalgzkmw	h. $h(x) = -\frac{3}{2}(x-3)^2 + 4$ mirrored over the $x-$ axis, vertical compression/horizontal expansion by a factor of $^3/_2$, right 3, up 4 https://www.desmos.com/calculator/mrwa6prrik
i. $y = 0.6(x+2)^2 + 5$ vertical expansion/horizontal compression by a factor of $^5/_3$, left 2, up 5 https://www.desmos.com/calculator/u1nbwidben	j. $j(x) = -\frac{1}{4}(x+4)^2 - 3$ mirrored over the $x-$ axis, vertical expansion/horizontal compression by a factor of 4, left 4, down 3 https://www.desmos.com/calculator/yckullvclh
k. $k(x) = 4(x-4)^2 - 2$ vertical compression/horizontal expansion by a factor of 4, right 4, down 2 https://www.desmos.com/calculator/bs3daor9nf	l. $y = -\frac{5}{4}(x-2)^2 + 6$ mirrored over the $x-$ axis, vertical compression/horizontal expansion by a factor of $^5/_4$, right 2, up 6 https://www.desmos.com/calculator/wsnouwebsw

2. Determine the equation of the following quadratic functions.

Developing		
a. $f(x) = (x+1)^2 - 2$	b. $(x-3)^2$	c. $f(x) = x^2 - 4$
Proficient		
d. $f(x) = \frac{1}{2}(x+2)^2 - 1$	e. $f(x) = 3(x-1)^2 - 4$	f. $f(x) = -2x^2 + 3$
g. $f(x) = -\frac{2}{3}(x+1)^2 + 2$	h. $h(x) = \frac{1}{4}(x-2)^2 + 2$	i. $f(x) = \frac{3}{2}(x+3)^2 - 3$

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For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 3.2	Given a quadratic function, identify the characteristics of graphs, including domain, range, intercepts, vertex and the axis of symmetry.

1. For the following equations, determine the $x-$ and $y-$ intercepts, the coordinates of the vertex, the equation of the axis of symmetry, the maximum or minimum value and the domain and range of the function.

Developing

a.

$$a(x) = x^2 - 4x - 5$$
$$= (x-5)(x+1)$$

$x-$ int.	$x = -1, 5$
$y-$ int.	$y = -5$
Vertex	$(2, -9)$
Axis of Symmetry	$x = 2$
Max/Min? Value?	Min at $y = -9$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -9, y \in \mathbb{R}\}$

b.

$$b(x) = x^2 - 10x + 24$$
$$= (x-4)(x-6)$$

$x-$ int.	$x = 4, 6$
$y-$ int.	$y = 24$
Vertex	$(5, -1)$
Axis of Symmetry	$x = 5$
Max/Min? Value?	Min at $y = -1$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -1, y \in \mathbb{R}\}$

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c.

$$c(x) = x^2 + 8x + 15$$
$$= (x + 5)(x + 3)$$

$x-$ int.	$x = -5, -3$
$y-$ int.	$y = 15$
Vertex	$(-4, -1)$
Axis of Symmetry	$x = -4$
Max/Min? Value?	Min at $y = -1$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -1, y \in \mathbb{R}\}$

d.

$$d(x) = x^2 + 2x - 35$$
$$= (x + 7)(x - 5)$$

$x-$ int.	$x = -7, 5$
$y-$ int.	$y = -35$
Vertex	$(-1, -36)$
Axis of Symmetry	$x = -1$
Max/Min? Value?	Min at $y = -36$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -36, y \in \mathbb{R}\}$

e.

$$y = x^2 - 8x$$
$$= x(x - 8)$$

$x-$ int.	$x = -1, 5$
$y-$ int.	$y = -5$
Vertex	$(2, -9)$
Axis of Symmetry	$x = 2$
Max/Min? Value?	Min at $y = -9$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -1, y \in \mathbb{R}\}$

f.

$$y = x^2 + 2x - 24$$
$$= (x + 6)(x - 4)$$

$x-$ int.	$x = -6, 4$
$y-$ int.	$y = -24$
Vertex	$(-1, -25)$
Axis of Symmetry	$x = -1$
Max/Min? Value?	Min at $y = -25$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -25, y \in \mathbb{R}\}$

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Proficient

g.

$$y = x^2 + 3x + 2$$
$$= (x + 2)(x + 1)$$

$x-$ int.	$x = -2, -1$
$y-$ int.	$y = 2$
Vertex	$\left(-\frac{3}{2}, -\frac{1}{4}\right)$
Axis of Symmetry	$x = -\frac{3}{2}$
Max/Min? Value?	Min at $y = -\frac{1}{4}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{1}{4}, y \in \mathbb{R}\}$

h.

$$b(x) = x^2 - 11x + 24$$
$$= (x - 8)(x - 3)$$

$x-$ int.	$x = 3, 8$
$y-$ int.	$y = 24$
Vertex	$\left(\frac{11}{2}, -\frac{25}{4}\right)$
Axis of Symmetry	$x = \frac{11}{2}$
Max/Min? Value?	Min at $y = -\frac{25}{4}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{25}{4}, y \in \mathbb{R}\}$

i.

$$g(x) = x^2 + 7x + 10$$
$$= (x + 5)(x - 2)$$

$x-$ int.	$x = -5, 2$
$y-$ int.	$y = 10$
Vertex	$\left(-\frac{3}{2}, -\frac{49}{4}\right)$
Axis of Symmetry	$x = -\frac{3}{2}$
Max/Min? Value?	Min at $y = -\frac{49}{4}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{49}{4}, y \in \mathbb{R}\}$

j.

$$h(x) = -6x^2 + 18x + 12$$
$$= -6(x - 2)(x - 1)$$

$x-$ int.	$x = 1, 2$
$y-$ int.	$y = 12$
Vertex	$\left(\frac{3}{2}, \frac{3}{2}\right)$
Axis of Symmetry	$x = \frac{3}{2}$
Max/Min? Value?	Max at $y = \frac{3}{2}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{3}{2}, y \in \mathbb{R}\}$

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k.

$$y = 7x^2 - 14x + 7$$
$$= 7(x-1)^2$$

$x-$ int.	$x = 1$
$y-$ int.	$y = 7$
Vertex	$(1, 0)$
Axis of Symmetry	$x = 1$
Max/Min? Value?	Min at $y = 0$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq 0, y \in \mathbb{R}\}$

l.

$$j(x) = -x^2 - 8x - 16$$
$$= -(x+4)^2$$

$x-$ int.	$x = -4$
$y-$ int.	$y = -16$
Vertex	$(-4, 0)$
Axis of Symmetry	$x = -4$
Max/Min? Value?	Max at $y = 0$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq 0, y \in \mathbb{R}\}$

m.

$$y = x^2 + 3x - 18$$
$$= (x+6)(x-3)$$

$x-$ int.	$x = -6, 3$
$y-$ int.	$y = -18$
Vertex	$\left(-\frac{3}{2}, -\frac{81}{4}\right)$
Axis of Symmetry	$x = -\frac{3}{2}$
Max/Min? Value?	Min at $y = -\frac{81}{4}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{81}{4}, y \in \mathbb{R}\}$

n.

$$y = -5x^2 + 55x - 150$$
$$= -5(x-6)(x-5)$$

$x-$ int.	$x = 5, 6$
$y-$ int.	$y = -150$
Vertex	$\left(-\frac{3}{2}, \frac{5}{4}\right)$
Axis of Symmetry	$x = -\frac{3}{2}$
Max/Min? Value?	Max at $y = \frac{5}{4}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{5}{4}, y \in \mathbb{R}\}$

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Extending

o.

$$k(x) = 4x^2 + 9x + 5$$
$$= (x + 1)(4x + 5)$$

$x-$ int.	$x = -\frac{5}{4}, -1$
$y-$ int.	$y = 5$
Vertex	$\left(-\frac{9}{8}, -\frac{1}{16}\right)$
Axis of Symmetry	$x = -\frac{9}{8}$
Max/Min? Value?	Min at $y = -\frac{1}{16}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{1}{16}, y \in \mathbb{R}\}$

p.

$$j(x) = 7x^2 - 6x$$
$$= x(7x - 6)$$

$x-$ int.	$x = 0, \frac{6}{7}$
$y-$ int.	$y = 0$
Vertex	$\left(\frac{3}{7}, -\frac{9}{7}\right)$
Axis of Symmetry	$x = \frac{3}{7}$
Max/Min? Value?	Min at $y = -\frac{9}{7}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{9}{7}, y \in \mathbb{R}\}$

q.

$$k(x) = 3x^2 - 16x - 12$$
$$= (x - 6)(3x + 2)$$

$x-$ int.	$x = -5, 2$
$y-$ int.	$y = -12$
Vertex	$\left(\frac{8}{3}, -\frac{100}{3}\right)$
Axis of Symmetry	$x = \frac{8}{3}$
Max/Min? Value?	Min at $y = -\frac{100}{3}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{100}{3}, y \in \mathbb{R}\}$

r.

$$y = 6x^2 - 13x + 6$$
$$= (3x - 2)(2x - 3)$$

$x-$ int.	$x = \frac{2}{3}, \frac{3}{2}$
$y-$ int.	$y = 6$
Vertex	$\left(\frac{13}{12}, -\frac{25}{24}\right)$
Axis of Symmetry	$x = \frac{13}{12}$
Max/Min? Value?	Min at $y = -\frac{25}{24}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{25}{24}, y \in \mathbb{R}\}$

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s.

$$y = 7x^2 + 2x$$
$$= x(7x + 2)$$

$x-$ int.	$x = -\frac{2}{7}, 0$
$y-$ int.	$y = 0$
Vertex	$\left(-\frac{1}{7}, -\frac{1}{7}\right)$
Axis of Symmetry	$x = -\frac{1}{7}$
Max/Min? Value?	Min at $y = -\frac{1}{7}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{1}{7}, y \in \mathbb{R}\}$

t.

$$y = 3x^2 - 2x - 5$$
$$= (3x - 5)(x + 1)$$

$x-$ int.	$x = -1, \frac{5}{3}$
$y-$ int.	$y = -5$
Vertex	$\left(\frac{1}{3}, -\frac{16}{3}\right)$
Axis of Symmetry	$x = \frac{1}{3}$
Max/Min? Value?	Min at $y = -\frac{16}{3}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{16}{3}, y \in \mathbb{R}\}$

u.

$$y = 8x - 3x^2 - 4$$
$$= -(x - 2)(3x - 2)$$

$x-$ int.	$x = \frac{2}{3}, 2$
$y-$ int.	$y = -4$
Vertex	$\left(\frac{4}{3}, \frac{4}{3}\right)$
Axis of Symmetry	$x = \frac{4}{3}$
Max/Min? Value?	Max at $y = \frac{4}{3}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{4}{3}, y \in \mathbb{R}\}$

v.

$$y = 2x^2 + 11x + 5$$
$$= (x + 5)(2x + 1)$$

$x-$ int.	$x = -5, -\frac{1}{2}$
$y-$ int.	$y = 5$
Vertex	$\left(-\frac{11}{4}, -\frac{81}{8}\right)$
Axis of Symmetry	$x = -\frac{11}{4}$
Max/Min? Value?	Min at $y = -\frac{81}{8}$
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{81}{8}, y \in \mathbb{R}\}$

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Extending
2. A bridge spans a horizontal distance of 40 metres and has a parabolic arch. One metre from the end of the bridge, the arch is 1.95 metres high. a. Determine an equation that represents this parabolic arch. $$y = -\frac{1}{20}x(x-40)$$ b. How high is the arch at the centre of the bridge? $$20 \text{ m}$$
3. A doorway is a parabolic arch. The base of the doorway is 2 metres wide. At a point 0.5 metres from one side of the doorway, it is 3 metres high. a. Determine an equation that represents this parabolic arch. $$y = -4x^2 + 8x$$ b. Could an item that is 3.5 metres high fit through the doorway? Justify your answer. Yes
4. The cross section of a satellite dish is parabolic. The parabola has a maximum depth of 7.5 cm and a width of 50 cm. a. Determine an equation to model the parabolic dish. $$y = \frac{3}{250}x(x-50)$$ b. How deep is the dish 10 cm from its edge? $$4.8 \text{ cm}$$
5. A student club us planning a fundraising car wash. Last year they charged $\$10$ per vehicle and washed 120 vehicles. They would like to earn more money this year. For every $\$1$ increase in price, they know they will wash 5 fewer vehicles. a. Write a quadratic function to model this situation. $$R(x) = 1200 + 70x - 5x^2$$ b. Determine the best price to charge for the car wash and the revenue expected at that price. $$\$17, R(17) = \$1445$$
6. A trough has a parabolic cross section. It is 2 metres wide at the top. At a point 0.75 metres from its top edge, the trough is 0.5 metres deep. Determine an equation that represents the cross section of the trough. What is the greatest depth of the trough? $$c(x) = \frac{8}{15}x(x-2)$$ $$c \approx 0.53 \text{ metres}$$

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For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 3.3	Convert standard form of the quadratic equation to vertex form by completing the square.

1. Convert the following equations to vertex form, then find the domain, range, vertex, $y-$ intercept, axis of symmetry, number of $x-$ intercepts and the maximum or minimum value.

Developing

a. $a(x) = x^2 - 6x + 8$
$= (x-3)^2 - 1$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -1, y \in \mathbb{R}\}$
Vertex	$(3, -1)$
$y-$ int.	$y = 8$
Axis of Symmetry	$x = 3$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -1$

b. $b(x) = x^2 + 10x + 14$
$= (x+5)^2 - 11$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -11, y \in \mathbb{R}\}$
Vertex	$(-5, -11)$
$y-$ int.	$y = 14$
Axis of Symmetry	$x = -5$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -11$

c. $c(x) = x^2 - 6x + 10$
$= (x-3)^2 + 1$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq 1, y \in \mathbb{R}\}$
Vertex	$(3, 1)$
$y-$ int.	$y = 10$
Axis of Symmetry	$x = 3$
$x-$ int.	0 int.
Max/Min? Value?	Min at $y = 1$

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d. $d(x) = x^2 + 6x + 5$
$= (x + 3)^2 - 4$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -4, y \in \mathbb{R}\}$
Vertex	$(-3, -4)$
$y-$ int.	$y = 5$
Axis of Symmetry	$x = -3$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -4$

e. $y = x^2 - 8x + 10$
$= (x - 4)^2 - 6$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -6, y \in \mathbb{R}\}$
Vertex	$(4, -6)$
$y-$ int.	$y = 10$
Axis of Symmetry	$x = 4$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -6$

f. $f(x) = x^2 + 10x - 2$
$= (x + 5)^2 - 27$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -27, y \in \mathbb{R}\}$
Vertex	$(-5, -27)$
$y-$ int.	$y = -2$
Axis of Symmetry	$x = -5$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -27$

g. $g(x) = x^2 - 2x + 4$
$= (x - 1)^2 + 3$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq 3, y \in \mathbb{R}\}$
Vertex	$(1, 3)$
$y-$ int.	$y = 4$
Axis of Symmetry	$x = 1$
$x-$ int.	0 int.
Max/Min? Value?	Min at $y = 3$

h. $h(x) = x^2 + 6x - 3$
$= (x + 3)^2 - 12$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -12, y \in \mathbb{R}\}$
Vertex	$(-3, -12)$
$y-$ int.	$y = -3$
Axis of Symmetry	$x = -3$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -12$

i. $y = x^2 + 4x + 15$
$= (x + 2)^2 + 11$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq 11, y \in \mathbb{R}\}$
Vertex	$(-2, 11)$
$y-$ int.	$y = 15$
Axis of Symmetry	$x = -2$
$x-$ int.	0 int.
Max/Min? Value?	Min at $y = 11$

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j. $j(x) = x^2 - 4x + 14$
$= (x-2)^2 + 10$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq 10, y \in \mathbb{R}\}$
Vertex	$(2, 10)$
$y-$ int.	$y = 14$
Axis of Symmetry	$x = 2$
$x-$ int.	0 int.
Max/Min? Value?	Min at $y = 10$

k. $k(x) = x^2 - 10x + 1$
$= (x-5)^2 - 24$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -24, y \in \mathbb{R}\}$
Vertex	$(5, -24)$
$y-$ int.	$y = 1$
Axis of Symmetry	$x = 5$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -24$

l. $y = x^2 - 12x + 20$
$= (x-6)^2 - 16$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -16, y \in \mathbb{R}\}$
Vertex	$(6, -16)$
$y-$ int.	$y = 20$
Axis of Symmetry	$x = 6$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -16$

Proficient

a. $a(x) = x^2 - 5x + 4$
$= \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{9}{4}, y \in \mathbb{R}\}$
Vertex	$\left(\frac{5}{2}, -\frac{9}{4}\right)$
$y-$ int.	$y = 4$
Axis of Symmetry	$x = \frac{5}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -{}^{9}/_{4}$

b. $b(x) = x^2 - 3x + 8$
$= \left(x - \frac{3}{2}\right)^2 + \frac{23}{4}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq \frac{23}{4}, y \in \mathbb{R}\}$
Vertex	$\left(\frac{3}{2}, \frac{23}{4}\right)$
$y-$ int.	$y = 8$
Axis of Symmetry	$x = \frac{3}{2}$
$x-$ int.	0 int.
Max/Min? Value?	Min at $y = {}^{23}/_{4}$

c. $c(x) = x^2 + 9x - 2$
$= \left(x + \frac{9}{2}\right)^2 - \frac{89}{4}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{89}{4}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{9}{2}, -\frac{89}{4}\right)$
$y-$ int.	$y = -2$
Axis of Symmetry	$x = -\frac{9}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -{}^{89}/_{4}$

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d. $d(x) = x^2 + 13x + 25$

$= \left(x + \frac{13}{2}\right)^2 - \frac{69}{4}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{69}{4}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{13}{2}, -\frac{69}{4}\right)$
$y-$ int.	$y = 25$
Axis of Symmetry	$x = -\frac{13}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -{}^{69}/_{4}$

e. $y = x^2 + 11x - 1$

$= \left(x + \frac{11}{2}\right)^2 - \frac{125}{4}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{125}{4}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{11}{2}, -\frac{125}{4}\right)$
$y-$ int.	$y = -1$
Axis of Symmetry	$x = -\frac{11}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -{}^{125}/_{4}$

f. $f(x) = x^2 - 7x + 10$

$= \left(x - \frac{7}{2}\right)^2 - \frac{9}{4}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{9}{4}, y \in \mathbb{R}\}$
Vertex	$\left(\frac{7}{2}, -\frac{9}{4}\right)$
$y-$ int.	$y = 10$
Axis of Symmetry	$x = \frac{7}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -{}^{9}/_{4}$

g. $g(x) = -x^2 + 4x + 5$

$= -(x-2)^2 + 9$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq 9, y \in \mathbb{R}\}$
Vertex	$(2, 9)$
$y-$ int.	$y = 5$
Axis of Symmetry	$x = 2$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = 9$

h. $h(x) = -x^2 + 8x - 1$

$= -(x-4)^2 + 15$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq 15, y \in \mathbb{R}\}$
Vertex	$(4, 15)$
$y-$ int.	$y = -1$
Axis of Symmetry	$x = 4$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = 15$

i. $y = -x^2 - 9x + 15$

$= -\left(x + \frac{9}{2}\right)^2 + \frac{141}{4}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{141}{4}, y \in \mathbb{R}\}$
Vertex	$\left(\frac{9}{2}, \frac{141}{4}\right)$
$y-$ int.	$y = 15$
Axis of Symmetry	$x = \frac{9}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = {}^{141}/_{4}$

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j. $j(x) = -x^2 - 5x - 3$

$= -\left(x + \frac{5}{2}\right)^2 + \frac{13}{4}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{13}{4}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{5}{2}, \frac{13}{4}\right)$
$y-$ int.	$y = -3$
Axis of Symmetry	$x = -\frac{5}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = {}^{13}/_{4}$

k. $k(x) = -x^2 - 12x + 2$

$= -(x + 6)^2 + 38$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq 38, y \in \mathbb{R}\}$
Vertex	$(-6, 38)$
$y-$ int.	$y = 2$
Axis of Symmetry	$x = -6$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = 38$

l. $y = -x^2 + 15x + 30$

$= -\left(x - \frac{15}{2}\right)^2 + \frac{345}{4}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{345}{4}, y \in \mathbb{R}\}$
Vertex	$\left(\frac{15}{2}, \frac{345}{4}\right)$
$y-$ int.	$y = 30$
Axis of Symmetry	$x = \frac{15}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = {}^{345}/_{4}$

m. $m(x) = 2x^2 + 14x + 5$

$= 2\left(x + \frac{7}{2}\right)^2 - \frac{39}{2}$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{39}{2}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{7}{2}, -\frac{39}{2}\right)$
$y-$ int.	$y = 5$
Axis of Symmetry	$x = -\frac{7}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -{}^{39}/_{2}$

n. $n(x) = -3x^2 + 12x - 3$

$= -3(x - 2)^2 + 9$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq 9, y \in \mathbb{R}\}$
Vertex	$(2, 9)$
$y-$ int.	$y = -3$
Axis of Symmetry	$x = 2$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = 9$

o. $y = 4x^2 + 12x - 3$

$= 4\left(x + \frac{3}{2}\right)^2 - 12$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -12, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{3}{2}, -12\right)$
$y-$ int.	$y = -3$
Axis of Symmetry	$x = -\frac{3}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -12$

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p. $p(x) = -5x^2 - 20x + 5$
$= -5(x+2)^2 + 25$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq 25, y \in \mathbb{R}\}$
Vertex	$(-2, 25)$
$y-$ int.	$y = 5$
Axis of Symmetry	$x = -2$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = 25$

q. $q(x) = 3x^2 + 6x + 10$
$= 3(x+1)^2 + 7$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq 7, y \in \mathbb{R}\}$
Vertex	$(-1, 7)$
$y-$ int.	$y = 10$
Axis of Symmetry	$x = -1$
$x-$ int.	0 int.
Max/Min? Value?	Min at $y = 7$

r. $r(x) = -6x^2 + 24x + 10$
$= -6(x-2)^2 + 34$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq 34, y \in \mathbb{R}\}$
Vertex	$(2, 34)$
$y-$ int.	$y = 10$
Axis of Symmetry	$x = 2$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = 34$

Extending

w.

$$a(x) = 2x^2 + 9x + 5$$
$$= 2\left(x + \frac{9}{4}\right)^2 - \frac{61}{4}$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{61}{4}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{9}{4}, -\frac{61}{4}\right)$
$y-$ int.	$y = 5$
Axis of Symmetry	$x = -\frac{9}{4}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -{}^{61}/_{4}$

x.

$$b(x) = -\frac{1}{2}x^2 + 3x + 8$$
$$= -\frac{1}{2}(x-3)^2 + \frac{25}{2}$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{25}{2}, y \in \mathbb{R}\}$
Vertex	$\left(3, \frac{25}{2}\right)$
$y-$ int.	$y = 8$
Axis of Symmetry	$x = 3$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = -{}^{25}/_{2}$

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y.

$$c(x) = \frac{1}{4}x^2 + 3x + 2$$
$$= \frac{1}{4}(x+6)^2 - 7$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -7, y \in \mathbb{R}\}$
Vertex	$(-6, -7)$
$y-$ int.	$y = 2$
Axis of Symmetry	$x = -6$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -7$

z.

$$d(x) = -\frac{3}{2}x^2 - 9x - 5$$
$$= -\frac{3}{2}(x+3)^2 + \frac{17}{2}$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{17}{2}, y \in \mathbb{R}\}$
Vertex	$\left(-3, \frac{17}{2}\right)$
$y-$ int.	$y = -5$
Axis of Symmetry	$x = -3$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = {}^{17}/_{2}$

aa.

$$y(x) = 3x^2 - 9x - 15$$
$$= 3\left(x - \frac{3}{2}\right)^2 - \frac{87}{4}$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{87}{4}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{3}{2}, -\frac{87}{4}\right)$
$y-$ int.	$y = -15$
Axis of Symmetry	$x = -\frac{3}{2}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -{}^{87}/_{4}$

bb.

$$f(x) = -\frac{4}{5}x^2 - 8x + 6$$
$$= -\frac{4}{5}(x+5)^2 + 26$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq 26, y \in \mathbb{R}\}$
Vertex	$(-5, 26)$
$y-$ int.	$y = 6$
Axis of Symmetry	$x = -5$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = 26$

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cc.

$$g(x) = \frac{5}{3}x^2 + 20x - 4$$
$$= \frac{5}{3}(x+6)^2 - 49$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -49, y \in \mathbb{R}\}$
Vertex	$(-6, -49)$
$y-$ int.	$y = -4$
Axis of Symmetry	$x = -6$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -49$

dd.

$$h(x) = -4x^2 - 10x + 12$$
$$= -4\left(x + \frac{5}{4}\right)^2 + \frac{73}{4}$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{73}{4}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{5}{4}, \frac{73}{4}\right)$
$y-$ int.	$y = 12$
Axis of Symmetry	$x = -\frac{5}{4}$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = {}^{73}/_{4}$

ee.

$$y = 5x^2 + 12x - 7$$
$$= 5\left(x + \frac{6}{5}\right)^2 - \frac{71}{5}$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq -\frac{71}{5}, y \in \mathbb{R}\}$
Vertex	$\left(\frac{6}{5}, -\frac{71}{5}\right)$
$y-$ int.	$y = -7$
Axis of Symmetry	$x = -\frac{6}{5}$
$x-$ int.	2 int.
Max/Min? Value?	Min at $y = -{}^{71}/_{5}$

ff.

$$j(x) = -2x^2 + 15x - 3$$
$$= -2\left(x - \frac{15}{4}\right)^2 + \frac{213}{4}$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{213}{4}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{15}{4}, \frac{213}{4}\right)$
$y-$ int.	$y = -3$
Axis of Symmetry	$x = -\frac{15}{4}$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = {}^{213}/_{4}$

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gg.

$$k(x) = 4x^2 + 2x + 1$$
$$= 4\left(x + \frac{1}{4}\right)^2 + \frac{3}{4}$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq \frac{3}{4}, y \in \mathbb{R}\}$
Vertex	$\left(-\frac{1}{4}, \frac{3}{4}\right)$
$y-$ int.	$y = 1$
Axis of Symmetry	$x = -\frac{1}{4}$
$x-$ int.	0 int.
Max/Min? Value?	Min at $y = {}^{3}/_{4}$

hh.

$$y = -4x^2 + x + 1$$
$$= -4\left(x - \frac{1}{8}\right)^2 + \frac{17}{16}$$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \leq \frac{17}{16}, y \in \mathbb{R}\}$
Vertex	$\left(\frac{1}{8}, \frac{17}{16}\right)$
$y-$ int.	$y = 1$
Axis of Symmetry	$x = \frac{1}{8}$
$x-$ int.	2 int.
Max/Min? Value?	Max at $y = {}^{17}/_{16}$

Extending

2. The profit, p, earned from the sale of a particular product by a business is given by

$$p = -0.25d^2 + 5d + 80$$

where d is the number of days the product has been for sale.

a. Determine the vertex of the profit function.

$$(10, 105)$$

b. Explain what the vertex means in the context of this problem.

When the product has been for sale for 10 days, the maximum profit of \$105 has been earned.

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3. The fuel consumption for a vehicle is related to the speed that it is driven and is usually given in litres per one hundred kilometres. Engines are generally more efficient at higher speeds than at lower speeds. For a particular type of car driving at a constant speed, the fuel consumption, C, in litres per one hundred kilometres, is related to the average driving speed, v, in kilometres per hour, by the function $C(v) = 0.004v^2 - 0.62v + 30$.
   a. Without graphing, determine the most efficient speed at which this car should be driven. Explain/show the strategy you use.

$$v = 77.5 \ {}^{\text{km}}/_{\text{h}}$$

   b. Describe any characteristics of the graph that you can identify without graphing and explain how you know.

Minimum Fuel Consumption	$\approx 6.0 \ {}^{\text{L}}/_{100 \text{ km}}$
No negative fuel consumption	
The vehicle uses $30 \ {}^{\text{L}}/_{100 \text{ km}}$ when it is not moving.	
Domain	$\{v \mid v \geq 0, v \in \mathbb{R}\}$ No negative speed
Range	$\{C \mid C \geq 6.0 \, , C \in \mathbb{R}\}$