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Chapter 3a Review
Derivatives

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 3.1	Using all basic derivative rules.

1. Find the following derivatives.

Developing		
a. $f(x) = x^{100}$ $f'(x) = 100x^{99}$	b. $f(t) = t^{-100}$ $f'(t) = -\dfrac{100}{t^{101}}$	c. $g(x) = \dfrac{1}{x^5}$ $g'(x) = -\dfrac{5}{x^6}$
d. $f(x) = x^{\pi}$ $f'(x) = \pi x^{\pi-1}$	e. $h(x) = x^{3/4}$ $h'(x) = \dfrac{3\sqrt[4]{x^3}}{4x}$	f. $g(s) = s^{-9/7}$ $g'(s) = -\dfrac{9}{7s^{16/7}}$
g. $f(x) = 7x^3$ $f'(x) = 21x^2$	h. $h(x) = 12x^{4/3}$ $h'(x) = 16\sqrt[3]{x}$	i. $g(x) = 3x^2 - 5\sqrt{x}$ $g'(x) = 6x - \dfrac{5\sqrt{x}}{2x}$
j. $y = (3x+2)^2$ $\dfrac{dy}{dx} = 18x + 12$	k. $h(x) = \left(\dfrac{x}{2}\right)^4$ $h'(x) = \dfrac{x^3}{8}$	l. $g(x) = 5(x^2)^4$ $g'(x) = 40x^7$
m. $s = t^2(t^2 - 2t)$ $\dfrac{ds}{dt} = 2t^2(2t-3)$	n. $y = 4x^{-1/2} - \dfrac{6}{x}$ $\dfrac{dy}{dx} = \dfrac{6 - 2\sqrt{x}}{x^2}$	o. $s(t) = \dfrac{t^5 - 3t^2}{2t}$ $s'(t) = \dfrac{1}{2}(4t^3 - 3)$
p. $f(x) = 5x^3 + 12x^2 - 15$ $f'(x) = 15x^2 + 24x$	q. $f(s) = -5s^5 + 3s^2 - \dfrac{5}{s^2}$ $f'(s) = -25s^4 + 6s + \dfrac{10}{s^3}$	r. $f(x) = 5(-3x^2 + 5x + 1)$ $f'(x) = 5(-6x + 5)$
s. $f(t) = (t+1)(t^2 + 2t - 3)$ $f'(t) = 3t^2 + 6t - 1$	t. $h(x) = (x+1)(x^2 + 2x - 3)^{-1}$ $h'(x) = -\dfrac{x^2 + 2x + 5}{(x^2 + 2x - 3)^2}$	u. $g(x) = x^3(x^3 - 5x + 10)$ $g'(x) = 2x^2(3x^3 - 10x + 15)$
v. $g(s) = (s^2 + 5s - 3)(s^5)$ $g'(s) = s^4(7s^2 + 10s - 15)$	w. $f(x) = (x^2 + 5x - 3)(x^{-5})$ $f'(x) = -\dfrac{3x^2 + 20x - 15}{x^6}$	x. $h(x) = (5x^3 + 12x^2 - 15)^{-1}$ $h'(x) = -\dfrac{3x(5x+8)}{(5x^3 + 12x^2 - 15)^2}$

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y. $y = 7x^4 - 7\pi^4 + \frac{1}{\pi\sqrt[3]{x}}$ $\frac{dy}{dx} = 28x^3 - \frac{1}{3\pi x^{4/3}}$	z. $f(x) = \frac{1-\sqrt{x}}{1+\sqrt{x}}$ $f'(x) = -\frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}$	aa. $y = \frac{(x-5)^2}{x^{20}}$ $\frac{dy}{dx} = \frac{-2(x-5)(9x-50)}{x^{21}}$
bb. $f(x) = \frac{x}{(x-625)^2}$ $f'(x) = -\frac{x+625}{(x-625)^3}$	cc. $t(w) = \frac{w^3}{w^3-5w+10}$ $t'(w) = -\frac{10w^2(w-3)}{(w^3-5w+10)^2}$	dd. $f(x) = 7 - 6\sqrt{x} + 5x^{2/3}$ $f'(x) = \frac{10\sqrt[3]{x^2} - 9\sqrt{x}}{3x}$
Proficient		
a. $y = (5x+1)^3(x-4)$ $y' = (5x+1)^2(8x-11)$	b. $y = (1-x^2)^4(2x+6)^3$ $\frac{dy}{dx} = -(3x^2-8x-27)$ $\times (2x+6)^2(1-x^2)^3$	c. $y = (3x^2+4)(3+x^3)^5$ $y' = (3+x^3)^4(6x^4+15x^2+18x$ $+ 20)$
d. $y = (x^2-9)^4(2x-1)^3$ $y' = 2(x^2-9)^3(2x-1)^2(11x^2$ $-4x-27)$	e. $f(x) = (x+1)^3(x+4)(x-3)^2$ $f'(x) = (x-3)(x+1)^2(6x^2$ $+11x-31)$	f. $y = x^2(3x^2+4)^2(3-x^3)^4$
g. $f(x) = (3-2x^3)^3$ $f'(x) = -18x^2(3-2x^3)^2$	h. $h(x) = x\sqrt{169-x^2}$ $h'(x) = \frac{\sqrt{169-x^2} - x^2}{\sqrt{169-x^2}}$	i. $f(t) = (t^2-4t+5)\sqrt{25-t^2}$ $f'(t)$ $= \frac{(2t-4)(25-t^2) - t(t^2-4t+5)}{\sqrt{25-t^2}}$
j. $f(x) = \sqrt{c^2-x^2}$ $f'(x) = \frac{-x}{\sqrt{c^2-x^2}}$	k. $y = \sqrt{1+x^4}$ $\frac{dy}{dx} = \frac{2x^3}{\sqrt{1+x^4}}$	l. $g(x) = \frac{1}{\sqrt{5-\sqrt{x}}}$ $g'(x) = \frac{1}{4\sqrt{x}\left(5-\sqrt{x}\right)^{3/2}}$
m. $f(x) = (1+3x)^2$ $f'(x) = 18x + 6$	n. $y = \frac{x^2+x+1}{1-x}$ $y' = -\frac{x^2-2x-2}{(1-x)^2}$	o. $g(x) = \frac{\sqrt{25-x^2}}{x}$ $g'(x) = -\frac{25}{x^2\sqrt{25-x^2}}$
p. $f(x) = \sqrt{\frac{169}{x} - x}$ $f'(x) = -\frac{169+x^2}{2x^2\sqrt{\frac{169}{x} - x}}$	q. $h(x) = \sqrt{x^3 - x^2 - \frac{1}{x}}$ $h'(x) = \frac{3x^4 - 2x^3 + 1}{2x^2\sqrt{x^3 - x^2 - \frac{1}{x}}}$	r. $g(x) = \frac{100}{(100-x^2)^{3/2}}$ $g'(x) = \frac{300x}{(100-x^2)^{5/2}}$
s. $f(x) = \sqrt[3]{x+x^3}$ $f'(x) = \frac{1^{2}_{3x}}{3(x+x^3)^{2/3}}$	t. $h(x) = (x+8)^5$ $h'(x) = 5(x+8)^4$	u. $g(x) = 4(2x^2-x+3)^{-2}$ $g'(x) = -\frac{8(4x-1)}{(2x^2-x+3)^3}$

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v. $f(x) = \sqrt[3]{x + x^3}$ $f'(x) = \dfrac{1 + 3x^2}{3(x + x^3)^{2/3}}$	w. $h(x) = \dfrac{1}{1 + 1/x}$ $h'(x) = \dfrac{1}{(x + 1)^2}$	x. $g(x) = (3x^2 + 1)(2x - 4)^3$ $g'(x) = 6(2x - 4)^2(5x^2 - 4x + 1)$
y. $y = \dfrac{3x - 2}{\sqrt{2x + 1}}$	z. $f(x) = 2x\sqrt{x^2 + 1}$	aa. $g(x) = (1 - x^{-1})^{-1}$
bb. $y = \left(x + \dfrac{1}{x^2}\right)^{\sqrt{7}}$	cc. $f(\mu) = \dfrac{(x + \mu)^4}{x^4 + \mu^4}$	dd. $g(x) = (1 - x^{-1})^{-1}$

Extending	
a. $f(x) = \sqrt{(x^2 + 1)^2 + \sqrt{1 + (x^2 + 1)^2}}$ $f'(x) = \dfrac{2x(x^2 + 1) + \dfrac{x(x^2 + 1)}{\sqrt{1 + (x^2 + 1)^2}}}{\sqrt{(x^2 + 1)^2 + \sqrt{1 + (x^2 + 1)^2}}}$	b. $f(x) = \dfrac{1}{\sqrt[3]{x + \sqrt{x}}}$
c. $f(x) = \dfrac{x^2 - 1}{x\sqrt{x^2 + 1}}$ $f'(x) = \dfrac{2x^2\sqrt{x^2 + 1} - (x^2 + 1)\left(\dfrac{2x^2 + 1}{\sqrt{x^2 + 1}}\right)}{x^2(x^2 + 1)}$	d. $g(x) = \sqrt{2 + \dfrac{3}{\sqrt{x}}}$ $g'(x) = -\dfrac{3}{4x^{3/2}}\left(2 + \dfrac{3}{\sqrt{x}}\right)^{-1/2}$

2. Find the equation for the tangent line to the given function at the given point.

Proficient	
e. $f(x) = \dfrac{x^3}{4} - \dfrac{1}{x}$ $\quad x = -2$ $y = \dfrac{13}{4}x + 5$	f. $f(x) = 3x^2 - \pi^3$ $\quad x = 4$ $y = 24x - 48 - \pi^3$
g. $f(x) = (2x - 3)^2$ $\quad x = 2$ $y = 4x - 7$	h. $f(x) = \dfrac{x^2 - 4}{5 - x}$ $\quad x = 3$ $y = \dfrac{17x}{4} - \dfrac{41}{4}$
i. $f(x) = \dfrac{x - 2}{x^3 + 4x - 1}$ $\quad x = 1$	j. $f(x) = -x^3 + 3x^2 - 2$ $\quad x = 1$

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Chapter 3a Review

Derivatives

$y=\frac{11}{16}x-\frac{15}{16}$	$y=3x-3$
k. $y=\frac{1}{x^{-5}}$ $x=-1$ $y=5x+4$	l. $y=\sqrt{16x^3}$ $x=4$ $y=12x-16$
Extending	
a. $y=(x^3-5x+2)(3x^2-2x)$ $x=1$	b. $f(x)=(5x^2+9x-2)(-x^2+2x+3)$ $x=1$
c. $f(x)=\frac{x^3}{x^2+9}$ $x=1$	d. $g(x)=\frac{(x+1)(x+2)}{(x-1)(x-2)}$ $x=4$
e. $f(x)=\frac{x^3}{x^2-6}$ $x=3$	f. $f(x)=(5x^2+9x-2)(-x^2+2x+3)$ $x=1$
g. $f(x)=\frac{\sqrt[3]{x-2}}{(x^3+4x-1)^2}$ $x=1$ $y=\frac{23}{96}x-\frac{29}{96}$	h. $f(x)=(x^2-4x+5)\sqrt{25-x^2}$ $x=3$ $y=\frac{13x}{2}-\frac{23}{2}$

3. Find the point(s) on the given function where the slope is the given value.

Proficient	
a. $f(x)=x^3+1$ $m=12$ $(-2,-7)$ $(2,9)$	b. $f(x)=\frac{2}{3}x^3+x^2-12x+6$ $m=-12$ $(0,6)$ $\left(-1,\frac{55}{3}\right)$
c. $f(x)=\frac{2}{3}x^3+x^2-12x+6$ $m=0$ $(-3,33)$ $\left(2,-\frac{26}{3}\right)$	d. $f(x)=(x^2+6)(x-5)$ $m=-2$ $\left(\frac{4}{3},-\frac{770}{27}\right)$ $(2,-30)$
e. $f(x)=-x^3+3x^2-2$ $m=0$ $(0,-2)$ $(2,2)$	f.
Extending	
a. $f(x)=\frac{2x+8}{\sqrt{x}}$ $m=0$ $(4,8)$	b. $f(x)=\frac{3x}{x-3}$ $m=-\frac{12}{25}$

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4. Find a cubic polynomial whose graph has horizontal tangents at $(-2, 5)$ and $(2, 3)$.

$$y = \frac{x^3}{16} - \frac{3x}{4} + 4$$

5. Do the functions $y = {}^{1}\!/_{x}$ and $y = x^3$ ever have the same slope? If so, where?

No

6. Determine the equations of the tangents to the curve $y = 2x^2 + 3$ that pass through the points
   a. $(2, 3)$
   b. $(2, -7)$

7. Show that there are no tangents to the following graph that have a negative slope.

$$f(x) = \frac{5x + 2}{x + 2}$$

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Learning Goal 3.2	Applying derivatives to trigonometric and exponential functions.

1. Determine the value of the infinite limit.

Proficient		
a. $f(x) = \sin x \cos x$ $f'(x) = \cos 2x$	b. $f(x) = \csc x - x \tan x$ $f'(x) = -\tan x - x\sec^2 x - \cot x \csc x$	c. $f(x) = 2\cos \pi x$ $f'(x) = -2\pi \sin \pi x$
d. $y = \sin(x^2+1)$ $\frac{dy}{dx} = 2x\cos(x^2+1)$	e. $g(x) = \tan 2x^2$ $g'(x) = 4x\sec^2 2x^2$	f. $h(x) = x\sin x$ $h'(x) = x\cos x + \sin x$
g. $f(x) = 2\sin 3x + 3\cos 2x$ $f'(x) = 6(\cos 3x - \sin 2x)$	h. $y = 2\cot 2x \ \sec 3x$ $\frac{dy}{dx} = -4\csc^2 2x + 3\tan 3x \sec 3x$	i. $g(x) = x^2 \cos 2x$ $g'(x) = 2x\cos x - x^2 \sin x$
j. $h(x) = \sin\sqrt{x^2-1}$ $h'(x) = \frac{x\cos\sqrt{x^2-1}}{\sqrt{x^2-1}}$	k. $f(x) = x\cos\frac{1}{x}$ $f'(x) = \frac{x\sin\left(\frac{1}{x}\right)}{x^2} + \cos\left(\frac{1}{x}\right)$	l. $g(x) = \frac{x - \sin x}{1 + \cos x}$
m. $h(x) = \sqrt{\tan x}$ $h'(x) = \frac{\sec^2 x}{2\sqrt{\tan x}}$	n. $f(x) = x^2 e^x$ $f'(x) = xe^x(x+2)$	o. $y = \frac{e^x}{1+x}$
p. $h(x) = e^{5x^3}$	q. $y = e^u(\cos u + 3u)$	r. $f(u) = e^{1/u}$
s. $g(x) = \sqrt{x}e^x$	t. $y = e^{x\sin 2x}$	u. $y = e^{2\tan\sqrt{x}}$
v. $f(a) = \sqrt{1+2e^{3a}}$	w. $f(x) = \cos(e^{\pi x})$	x. $f(x) = e^{e^x}$
y. $y = \cos x \tan x$	z. $f(x) = \sin(\cos x)$	aa. $f(x) = \tan\sqrt{1-x}$
bb. $g(c) = \sec(1+c^2)$	cc. $y = \sqrt{\sin\sqrt{x}}$	dd. $f(x) = \cot(3x^2+5)$

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ee. $g(x) = x^2 \sin x \cos x$ $g'(x) = 2x\sin x\cos x + x^2\cos^2 x - x^2\sin^2 x$	ff. $y = \dfrac{x\sin x}{1+\sin x}$ $y' = \dfrac{(\sin x + x\cos x)(1+\sin x) - x\sin x\cos x}{(1+\sin x)^2}$	gg. $y = \sin^3 x - \sin(x^3)$ $\dfrac{dy}{dx} = 3\sin^2 x\cos x - 3x^2\cos(x^3)$
Extending		
a. $y = \sqrt{1+xe^{-2x}}$	b. $f(x) = \dfrac{ae^x+b}{ce^x+d}$	c. $y = \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$
d. $y = \dfrac{1}{\sin(x-\sin x)}$	e. $f(\theta) = \dfrac{\sec 2\theta}{1+\tan 2\theta}$	f. $y = \sin(\tan\sqrt{1+x^3})$
g. $f(x) = \dfrac{\sin mx}{x}$	h. $f(\theta) = \tan^2(\sin\theta)$	i. $g(x) = \sqrt[5]{x\tan x}$
j. $y = \cos^2\left(\dfrac{1-x}{1+x}\right)$ $y' = \dfrac{4}{(1+x)^2}\cos\left(\dfrac{1-x}{1+x}\right)\sin\left(\dfrac{1-x}{1+x}\right)$	k. $h(x) = \tan(\sin(x^2+\sec^2 x))$ $h'(x) = (2x + 2\sec^2 x\tan x)\sec^2(\sin(x^2+\sec^2 x))\cos(x^2+\sec^2 x)$	l. $y = \dfrac{1}{2+\sin\left(\dfrac{\pi}{x}\right)}$ $y' = \dfrac{\pi\cos\left(\dfrac{\pi}{x}\right)}{x^2\left(2+\sin\left(\dfrac{\pi}{x}\right)\right)^2}$

2. Find the equation for the tangent line to the given function at the given point.

Extending	
a. $y = e^{ex}\cos \pi x$ $\quad x = 0$	b. $y = \dfrac{e^x}{x}$ $\quad x = 1$
c. $y = 4\sin^2 x$ $\quad x = \dfrac{\pi}{6}$	d. $y = \sqrt{1+4\sin x}$ $\quad x = 0$

3. Find the point(s) on the given function where the slope is the given value.

Extending	
a. $f(x) = x + 2\cos x$ $\quad m = 0$ $\dfrac{\pi}{6} + 2\pi n, n \in \mathbb{Z}$ $\dfrac{5\pi}{6} + 2\pi n, n \in \mathbb{Z}$	b. $y = \sin x + \cos x$ $\quad m = 0$