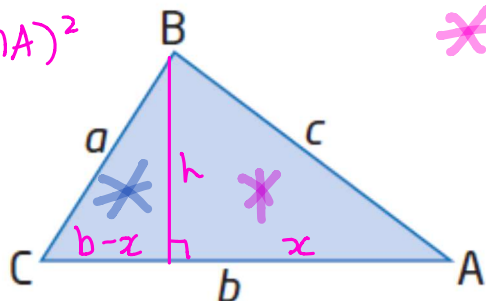


Name: _____

Date: _____

Learning Goal 2.3	Use of sine and cosine laws to solve non-right triangles, including ambiguous cases.
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$\sin^2 A = (\sin A)^2$



$c \times \sin A = \frac{h}{c} \times c$ $c \times \cos A = \frac{x}{c} \times c$

$h = c \times \sin A$

$x = c \times \cos A$

$a^2 = h^2 + (b-x)^2$

$a^2 = (c \times \sin A)^2 + (b - c \times \cos A)^2$

$a^2 = c^2 \sin^2 A + (b - c \times \cos A)(b - c \times \cos A)$

$a^2 = c^2 \sin^2 A + b^2 - 2bc \times \cos A + c^2 \cos^2 A$

$a^2 = c^2 (\sin^2 A + \cos^2 A) + b^2 - 2bc \times \cos A$

$\left(\frac{h}{c}\right)^2 + \left(\frac{x}{c}\right)^2$

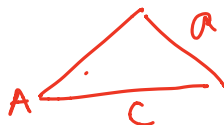
$= \frac{h^2}{c^2} + \frac{x^2}{c^2}$
 $= \frac{h^2 + x^2}{c^2}$
 $= \frac{c^2}{c^2} = 1$

Cosine Law

$a^2 = c^2 + b^2 - 2bc \times \cos A$

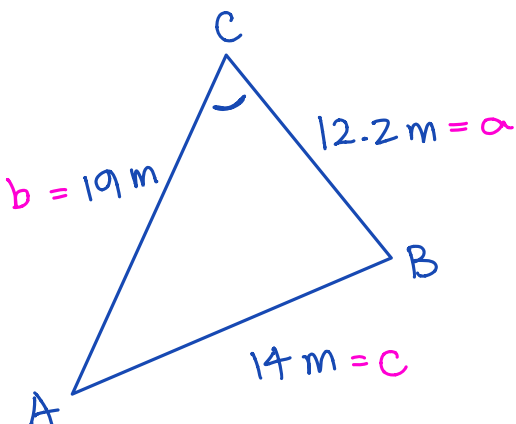
$b^2 = a^2 + c^2 - 2ac \times \cos B$

$c^2 = a^2 + b^2 - 2ab \times \cos C$



Sine Law versus Cosine Law	
When you're given a single angle-side pair - SSA - ASA	- only sides SSS - SAS (side-angle-side)

Example Lions Gate Bridge has been a Vancouver landmark since it opened in 1938. It is the longest suspension bridge in Western Canada. The bridge is strengthened by triangular braces. Suppose one brace has side lengths 14 m, 19 m and 12.2 m. Determine the measure of the angle opposite the 14 – m side to the nearest degree.



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$(14)^2 = (12.2)^2 + (19)^2 - 2(12.2)(19) \cos C$$

$$196 = 148.84 + 361 - 463.6 \cos C$$

$$-148.84 \quad -148.84 \quad -361$$

$$-361$$

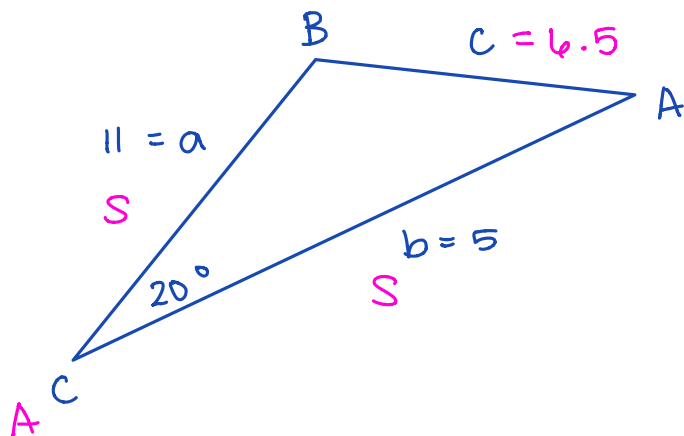
$$\underline{-313.84} = \underline{-463.6 \cos C}$$

$$\underline{-463.6} \quad \underline{-463.6}$$

$$\cos^{-1}(0.67696) = \underset{\cos^{-1}}{(\cos C)}$$

$$C \doteq 47^\circ$$

Example In $\triangle ABC$, $a = 11$, $b = 5$ and $\angle C = 20^\circ$. Sketch a diagram and determine the length of the unknown side and the measures of the unknown angles to the nearest tenth.



SAS \Rightarrow cosine law

$$\textcircled{1} c^2 = a^2 + b^2 - 2ab \cos C$$

$$= (11)^2 + (5)^2 - 2(11)(5) \cos 20$$

$$= 121 + 25 - 110 \cos 20$$

$$= 146 - 110 \cos 20$$

$$\sqrt{c^2} = \sqrt{42.6}$$

$$c = \underline{\underline{\pm 6.5}}$$

$$\textcircled{2} \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$5 \times \frac{\sin B}{5} = \frac{\sin 20}{6.5} \times 5$$

$$\sin B = \frac{5 \times \sin 20}{6.5}$$

$$\textcircled{3} A + B + C = 180^\circ$$

$$A + 15 + 20 = 180^\circ$$

$$A = 180^\circ - 20^\circ - 15^\circ$$

$$= 145^\circ$$

$$B = \sin^{-1}\left(\frac{5 \times \sin 20}{6.5}\right) \doteq 15^\circ$$