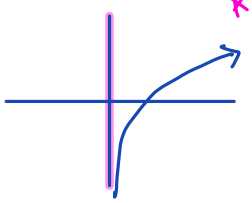
Name: _____ Date: _____

Learning Goal 2.2	Limits at infinity and the definition of the derivative

There are **three** different types of asymptotes:

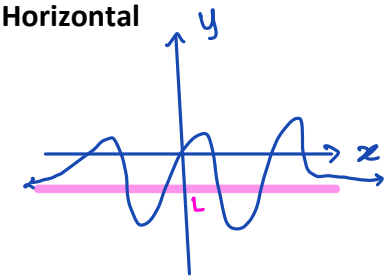
Vertical



* will never be crossed.

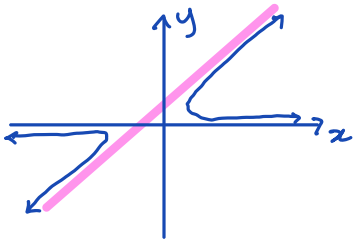
- reduce function and remaining zeros in the denominator are the vertical asymptotes.

Horizontal



- only when $x \to \pm\infty$ (no finite value)
- rational functions where
  - the degree numerator $\leq$ the degree of the denominator

Slant/Oblique



degree numerator = degree denominator + 1

- find using long/synthetic division

Example Find the asymptote equation(s) and type(s) of the following functions, if any.

a. $f(x) = \dfrac{0x^2+1}{x^2-x-6}$

$= \dfrac{1}{(x-3)(x+2)}$

HA @ $y = \frac{0}{1} = 0$

VA: $x = 3$, $x = -2$

b. $f(x) = \dfrac{x^4-5x^2+4}{x-1}$ $(x^2-4)(x^2-1)$

$= \dfrac{(x^2-4)(x+1)(x-1)}{x-1}$

NO asymptotes
Hole @ $x = 1$

c. $f(x) = \dfrac{x^2+8x-20}{x-1}$

* expect a slant asymptote

1	1	8	-20
	↓	1	9
	1	9	~~-11~~

VA: $x - 1 = 0$, $x = 1$

Slant asymptote @ $y = x + 9$

d. $f(x) = \dfrac{2x^2-4x+8}{3x^2-27}$ * VA OR HA

$\lim_{x \to \pm\infty} \dfrac{2x^2}{3x^2} = \dfrac{2}{3}$ Horizontal asymptote. $y = \frac{2}{3}$

$= \dfrac{2(x^2-2x+4)}{3(x^2-9)}$

↑ VA @ $x = \pm 3$

Assignment # 1 – 9, 13 – 21, 31, 41 – 47 Quiz Next Day!

e. $f(x) = \dfrac{1x - 7}{1x + 5}$

VA: $x+5=0$, $x=-5$

HA: $\lim\limits_{x\to\infty} \dfrac{x-7}{x+5} = \lim\limits_{x\to\infty} \dfrac{x}{x} = 1$ $\Rightarrow y = 1$

f. $f(x) = \dfrac{3 + 0x^2}{1x^2 - 2}$

VA: $x^2-2=0$, $x^2=2$, $x=\pm\sqrt{2}$

HA: $y = \frac{0}{1} = 0$

g. $f(x) = \dfrac{x^2}{x - 5}$

VA: $x-5=0$, $x=5$

SA: 5 | 1 0 0 / $\downarrow$ 5 25 / 1 5 ~~25~~

$y = x+5$

h. $f(x) = \dfrac{2x^2 - 5x + 3}{x - 1}$ ← looks like a SA - make sure to check first!

$= \dfrac{(x-1)(2x-3)}{x-1}$

$= 2x - 3$

Hole: $x-1=0$, $x=1$

i. $f(x) = \dfrac{7x^2 + 5x - 2}{2x^2 - 18}$

$= \dfrac{(7x-2)(x+1)}{2(x+3)(x-3)}$

VA: $x+3=0$, $x=-3$; $x-3=0$, $x=3$

HA: $y = \frac{7}{2}$

j. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2}$

VA: $x-2=0$, $x=2$

SA: 2 | 2 -5 5 / $\downarrow$ 4 -2 / 2 -1 ~~3~~

$y = 2x-1$

k. $f(x) = \dfrac{1}{3 - x}$

VA: $3-x=0$, $x=3$

l. $f(x) = \dfrac{x^2 - 4 + 0x^4}{x^4 - 81}$

$= \dfrac{(x+2)(x-2)}{(x^2+9)(x+3)(x-3)}$

VA: $x+3=0$, $x=-3$; $x-3=0$, $x=3$

HA: $y = \frac{0}{1} = 0$