

Name: _____

Date: _____

Learning Goal 5.3	Apply order of operations to radical expressions.
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Recall Multiplying polynomials

$$\begin{aligned} (5x^1)(8x^2) \\ = 40x^{1+2} \\ = 40x^3 \end{aligned}$$

$$\begin{aligned} 7y(6 - 9y) \\ = 42y - 63y^2 \end{aligned}$$

$$\begin{aligned} (z - 3)(z + 3) \\ = z^2 + 3z - 3z - 9 \\ = z^2 - 9 \end{aligned}$$

FOIL

↑ difference of squares

Example Multiply. Simplify the products where possible. State any restrictions on the variable, if any.

a. $(2\sqrt{7})(4\sqrt{75})$

$$\begin{aligned} \sqrt{75} &= \sqrt{3 \times 25} \\ &= 5\sqrt{3} \end{aligned}$$

$$\begin{aligned} &= (2\sqrt{7})(20\sqrt{3}) \\ &= 40\sqrt{21} \end{aligned}$$

b. $7\sqrt{3}(5\sqrt{5} - 6\sqrt{3})$

$$\begin{aligned} &= 35\sqrt{15} - 42\sqrt{9} \\ &= 35\sqrt{15} - 42 \times 3 \\ &= 35\sqrt{15} - 126 \end{aligned}$$

c. $(8\sqrt{2} - 5)(9\sqrt{5} + 6\sqrt{10})$

$$\begin{aligned} \sqrt{20} &= \sqrt{2^2 \times 5} = 2\sqrt{5} \\ &4 \quad 5 \\ &\wedge \\ &20 \end{aligned}$$

$$\begin{aligned} &= 72\sqrt{10} + 48\sqrt{20} - 45\sqrt{5} - 30\sqrt{10} \\ &= 72\sqrt{10} + 96\sqrt{5} - 45\sqrt{5} - 30\sqrt{10} \\ &= 42\sqrt{10} + 51\sqrt{5} \end{aligned}$$

d. $9\sqrt[3]{2w}(\sqrt[3]{4w} + 7\sqrt[3]{28})$

$$\begin{aligned} &= 9\sqrt[3]{8w^2} + 63\sqrt[3]{56w} \\ &= 18\sqrt[3]{w^2} - 126\sqrt[3]{7w} \end{aligned}$$

$\begin{matrix} \wedge \\ 2 \quad 28 \\ \hline 2 \quad 14 \\ \hline 2 \quad 7 \end{matrix}$

Example Divide. Simplify the products where possible. State any restrictions on the variable, if any.

a. $\frac{24\sqrt{x^2}}{\sqrt{3x}}$ ← radical: $x \geq 0$
 denominator: $x \neq 0$
 $\Rightarrow x > 0$

$$= \frac{24\sqrt{x}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{24\sqrt{3x}}{3}$$

$$= 8\sqrt{3x}$$

$\sqrt{3} \times \sqrt{3}$
 $= \sqrt{9}$
 $= 3$

b. $\frac{4\sqrt{5n}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$$= \frac{4\sqrt{10n}}{3 \times 2}$$

$$= \frac{2\sqrt{10n}}{3}$$

radical: $n \geq 0$

Rationalize
 Remove all radicals from the denominator

1. $\sqrt{5}$
 bump radical to the other term.

2. $\sqrt{5} + 7$
 $= (\sqrt{5} + 7)(\sqrt{5} + 7)$
 $= 5 + 7\sqrt{5} + 7\sqrt{5} + 49$

c. $\frac{11}{\sqrt{5} + 7} \times \frac{\sqrt{5} - 7}{\sqrt{5} - 7}$

$$= \frac{11\sqrt{5} - 77}{5 - 49}$$

$$= \frac{11\sqrt{5} - 77}{-44} = \frac{\sqrt{5} - 7}{-4} = \frac{7 - \sqrt{5}}{4}$$

d. $\frac{4\sqrt[3]{11}}{y\sqrt[3]{6}} \times \frac{\sqrt[3]{6^2}}{\sqrt[3]{6^2}}$ ← watch the index

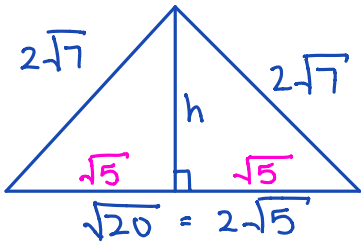
$$= \frac{4\sqrt[3]{6^2 \times 11}}{y\sqrt[3]{6^3}}$$

$$= \frac{4\sqrt[3]{6^2 \times 11}}{6y}$$

by

Conjugate
 A polynomial that will create a difference of squares

Example An isosceles triangle has a base of $\sqrt{20}$ metres. Each of the equal sides is $2\sqrt{7}$ metres long. What is the **exact** area of the triangle?



$$A = \frac{b \times h}{2} \Rightarrow \frac{2\sqrt{5} \times \sqrt{23}}{2}$$

$$= \sqrt{115}$$

$$a^2 + b^2 = c^2$$

$$(\sqrt{5})^2 + h^2 = (2\sqrt{7})^2$$

$$5 + h^2 = 28$$

$$-5 \quad -5$$

$$h^2 = 23 \Rightarrow h = \pm\sqrt{23}$$

The exact area of the triangle is $\sqrt{115}$ metres.