Name: _____ Date: _____

Learning Goal 0.2	**Expectations for algebra from previous years.**

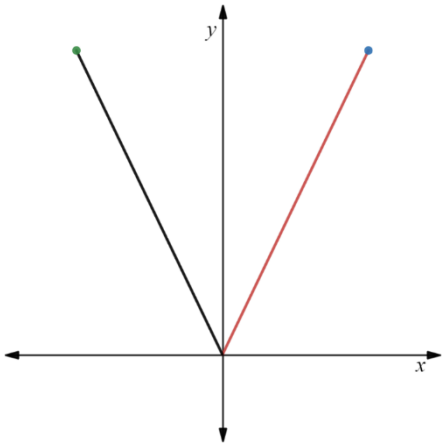
1. Solve the following trigonometric equation on the specified domain for exact value(s) of θ.

a. $4\csc x - 5 = 3, -180° \leq x < 360°$

$$4\csc x - 5 = 3$$
$$4\csc x = 8$$
$$\csc x = 2$$
$$\sin x = \frac{1}{2}$$
$$x = \sin^{-1}\left(\frac{1}{2}\right)$$
$$x_1 = 30°$$

$x_R = 30°$ $\qquad$ $x_2 = 180° - 30°$

$x_2 = 150°$

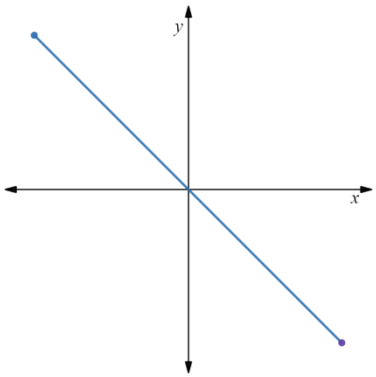


b. $7\cot\theta - 4 = 6\cot\theta - 5, 0 \leq \theta < 4\pi$

$$7\cot\theta - 4 = 6\cot\theta - 5$$
$$7\cot\theta = 6\cot\theta - 1$$
$$\cot\theta = -1$$
$$\tan\theta = -1$$
$$\theta_1 = \frac{3\pi}{4}$$
$$\theta_2 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$\theta_R = \frac{\pi}{4}$ $\qquad$ $\theta_3 = 3\pi - \frac{\pi}{4} = \frac{11\pi}{4}$

$$\theta_4 = 4\pi - \frac{\pi}{4} = \frac{15\pi}{4}$$



c. $\cos^2\theta - 1 = 0,\ \ 0° \le \theta < 360°$

$$\cos^2\theta - 1 = 0$$
$$\cos^2\theta = 1$$
$$\cos\theta = \pm 1$$

$\cos\theta_1 = 1$

$$\cos\theta = \frac{x}{r}$$
$$x = r$$
$$y = 0$$

$\theta_1 = 0°$

$\cos\theta_2 = -1$

$$\cos\theta = \frac{x}{r}$$
$$-x = r$$
$$y = 0$$

$\theta_2 = 180°$

d. $\sin^2\theta + \sin\theta - 2 = 0, 0 \le \theta < 2\pi$

$$\sin^2\theta + \sin\theta - 2 = 0$$
$$(\sin\theta + 2)(\sin\theta - 1) = 0$$

$\sin\theta_1 + 2 = 0$

$\sin\theta_1 = -2$

θ_1 DNE

$\sin\theta_2 - 1 = 0$

$\sin\theta_2 = 1$

$$\sin\theta = \frac{y}{r}$$
$$y = r$$
$$x = 0$$

$\theta_2 = \frac{\pi}{2}$

e. $\sec^2 x - 4 = 0, 0 \le x < 2\pi$

$$\sec^2 x - 4 = 0$$
$$\sec^2 x = 4$$
$$\sec x = \pm 2$$
$$\cos x = \pm\frac{1}{2}$$

$\cos x = \frac{1}{2}$

$$\cos x = \frac{x}{r}$$
$$x = 1$$
$$r = 2$$
$$x_R = \frac{\pi}{3}$$

$$x_1 = x_R = \frac{\pi}{3}$$

$$x_2 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$\cos x = -\frac{1}{2}$

$$\cos x = \frac{x}{r}$$
$$x = -1$$
$$r = 2$$
$$x_R = \frac{\pi}{3}$$

$$x_3 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$x_4 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

f. $3\tan^2 x - \tan x = 4, -\pi \leq x < 2\pi$

$$3\tan^2 x - \tan x = 4$$
$$3\tan^2 x - \tan x - 4 = 0$$
$$3\tan^2 x - 4\tan x + 3\tan x - 4 = 0$$
$$\tan x\,(3\tan x - 4) + (3\tan x - 4) = 0$$
$$(\tan x + 1)(3\tan x - 4) = 0$$

$\tan x + 1 = 0$
$\tan x = -1$

$\tan x = \dfrac{y}{x}$
$x = \pm 1$
$y = \pm 1$
$x_R = \dfrac{\pi}{4}$

$x_1 = -\dfrac{\pi}{4}$

$x_2 = \pi - \dfrac{\pi}{4}$
$= \dfrac{3\pi}{4}$

$x_3 = 2\pi - \dfrac{\pi}{4}$
$= \dfrac{7\pi}{4}$

$3\tan x - 4 = 0$
$3\tan x = 4$
$\tan x = \dfrac{4}{3}$

$\tan x = \dfrac{y}{x}$
$y = 4$
$x = 3$
$r = 5$

$x_4 = \tan^{-1}\left(\dfrac{4}{3}\right)$
$= 0.927$

$x_5 = \pi + x_4$
$= 4.069$

$x_6 = -\pi + x_4$
$= -2.214$

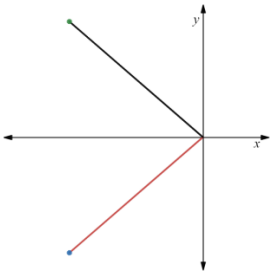
g. $4\cos^3 x + \cos 2x = 2\cos x$

$$4\cos^3 x + (2\cos^2 x - 1) = 2\cos x$$
$$4\cos^3 x + 2\cos^2 x - 2\cos x - 1 = 0$$
$$2\cos^2 x\,(2\cos x + 1) - (2\cos x + 1) = 0$$
$$(2\cos x + 1)(2\cos^2 x - 1) = 0$$

$2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$

$x_R = \frac{\pi}{3}$



$x_1 = \pi - \frac{\pi}{3}$ $\quad x_2 = \pi + \frac{\pi}{3}$

$x_1 = \frac{2\pi}{3}$ $\quad x_2 = \frac{4\pi}{3}$

$x_1 = \frac{2\pi}{3} + 2\pi n,\ n\epsilon\mathbb{Z}$

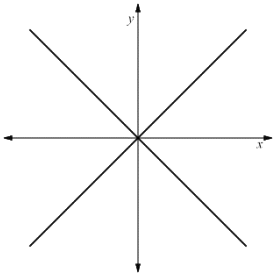
$x_2 = \frac{4\pi}{3} + 2\pi n,\ n\epsilon\mathbb{Z}$

$2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$x_R = \frac{\pi}{4}$



$x_3 = \frac{\pi}{4}$ $\quad x_4 = \pi - \frac{\pi}{4}$

$x_4 = \frac{3\pi}{4}$

$x_3 = \frac{\pi}{4} + \pi n,\ n\epsilon\mathbb{Z}$

$x_4 = \frac{3\pi}{4} + \pi n,\ n\epsilon\mathbb{Z}$