For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 4.1 Identify and order irrational numbers.

- Tell whether each number is rational or irrational.
- For those that are irrational, estimate the value of the radical to **one decimal place**, without a **calculator**.
- For those that are rational, evaluate the radical **without a calculator**. You may use your sheet of perfect numbers (from Section 4.1)

Developing	Pro	Proficient		
1. $\sqrt{81}$	2. ⁵ √100	3. $\sqrt[4]{2000}$		
rational	irrational	• irrational		
• = 9	 ≈ 2.5 	 ≈ 6.7 		
4. $\sqrt{169}$	5. $\sqrt[3]{64}$	6. $\sqrt[3]{1738}$		
rational	rational	irrational		
• = 13	• = 4	 ≈ 12.0 		
7. $\sqrt{150}$	8. $\sqrt[4]{16}$	9. $\sqrt[5]{1864}$		
irrational	rational	irrational		
 ≈ 12.2 	• = 2	• ≈ 4.5		
$10.\sqrt{125}$	11. $\sqrt[3]{81}$	12. $\sqrt[3]{100}$		
irrational	irrational	irrational		
 ≈ 11.2 	• ≈ 4.3	• ≈ 4.6		
13. $\sqrt{121}$	14. $\sqrt[3]{216}$	15 . ∜ <u>1 296</u>		
 rational 	rational	rational		
• = 11	• = 6	• = 6		
16. $\sqrt{49}$	17. ⁴ √75	18. $\sqrt[7]{128}$		
 rational 	irrational	rational		
• = 7	• ≈ 2.9	• = 2		
19. $\sqrt{200}$	20. ⁴ √ <u>81</u>	21 . ⁵ √248 832		
irrational	rational	rational		
 ≈ 14.1 	• = 3	• = 12		

Developing				
State the index and the radicand of each radical.				
1. $\sqrt[3]{64}$	2. $\sqrt[4]{20000}$			
3, 64	4, 20 000			
3. $\sqrt[4]{16}$	4. $\sqrt{1738}$			
4, 16	2, 1 738			

5. $\sqrt[3]{216}$	6. $\sqrt[5]{1864}$
3, 216	5, 1 864
7. ∜81	8. $\sqrt[3]{10000}$
4,81	3, 10 000

Proficient/Extending

 Use a number line to order these numbers from least to greatest, without a calculator.

 1.
$$\sqrt{25}$$
, $\sqrt[3]{30}$, $\sqrt[3]{-8}$, $\sqrt[4]{20}$, $\sqrt{\frac{144}{9}}$
 $\sqrt[3]{-8}$, $\sqrt[4]{20}$, $\sqrt[3]{30}$, $\sqrt{\frac{144}{9}}$, $\sqrt{25}$

 2. $\sqrt[5]{-243}$, $\sqrt{4}$, $\sqrt[3]{\frac{27}{216}}$, $\sqrt{12}$, $\sqrt[3]{6}$
 $\sqrt[5]{-243}$, $\sqrt{4}$, $\sqrt[3]{\frac{27}{216}}$, $\sqrt{12}$, $\sqrt[3]{6}$

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Learning Goal 4.2	Express an entire radical as a simplified mixed radical		
Learning Goal 4.2	and vice versa.		

Developing	Proficient	Extending
Write each radical in simplest form,	without a calculator.	
1. $\sqrt{125}$	2. $\sqrt[3]{216}$	3. $\sqrt{216k^3}$
$5\sqrt{5}$	6	$6k\sqrt{6k}$
4. $\sqrt{512}$	5. ⁴ √405	6. $\sqrt[3]{-96xy^4}$
$2^4\sqrt{2} = 16\sqrt{2}$	3∜√5	$2y\sqrt[3]{-12xy}$
7. √ <u>80</u>	8. $\sqrt[3]{-100}$	9. $\sqrt[4]{48m^5n^4}$
$4\sqrt{5}$	$\sqrt[3]{-100}$	$2mn\sqrt[4]{3m}$
$10.\sqrt{147}$	11. $\sqrt[3]{80}$	12. $\sqrt[5]{a^7b^{10}c^2d^{12}}$
$7\sqrt{3}$	$2\sqrt[3]{10}$	$ab^2d^2\sqrt[5]{a^2c^2d^2}$

Developing	Proficient	Extending		
Write each mixed radical as an entire radical, without a calculator.				
1. $2\sqrt{3}$	2. $-2\sqrt[3]{8}$	3. $12\sqrt{k^3}$		
$\sqrt{2^2 \times 3} = \sqrt{12}$	$\sqrt[3]{-2^3 \times 8} = \sqrt[3]{-64}$	$\sqrt{12^2k^3} = \sqrt{144k^3}$		
4. $3\sqrt{5}$	5. $5\sqrt[4]{2}$	6. $-xy\sqrt[3]{9xy^4}$		
$\sqrt{3^2 \times 5} = \sqrt{45}$	$\sqrt[4]{2 \times 5^4} = \sqrt[4]{1250}$	$\sqrt[3]{(-xy)^3 \times 9xy^4} = \sqrt[3]{-9x^4y^7}$		
7. $6\sqrt{3}$	8. $6\sqrt[3]{10}$	9. $-3m\sqrt[4]{4mn^4}$		
$\sqrt{3 \times 6^2} = \sqrt{108}$	$\sqrt[3]{6^3 \times 10} = \sqrt[3]{2 \ 160}$	$\sqrt[4]{(-3m)^2 \times 4mn^4} = \sqrt[4]{36m^3n^4}$		
10. $4\sqrt{2}$	11. $-8\sqrt[3]{8}$	12. $a^2 d^3 \sqrt[5]{a^3 b^5 c^2}$		
$\sqrt{2 \times 4^2} = \sqrt{32}$	$\sqrt[3]{8 \times -8^3} = \sqrt[3]{-4.096}$	$\sqrt[5]{(a^2d^3)^5 \times a^3b^5c^2}$		
		$= \sqrt[5]{a^{13}b^5c^2d^{15}}$		

Extending		
Simplify the radical.		
$\sqrt[n]{3 \times 2^n \times x^{2n} y^{n+3}}$		
$2x^2y\sqrt[n]{3y^3}$		
Write the mixed radical as an entire radical.		
$2ab^2 \times \sqrt[n]{5ab^2}$		
$\sqrt[n]{(2ab^2)^n \times 5ab^2} = \sqrt[n]{2^n \times 5 \times a^{n+1}b^{2n+2}}$		

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Learning Goal 4.3	Evaluate expressions with fractional and negative exponents. Connect fractional exponents to radicals, and negative
	exponents to reciprocals.

Developing				
Simplify each power, to a mixed radical where appropriate, without a calculator.				
1. $20^{1/2}$	2. $12^{1/2}$	3. $15^{1/3}$		
2\\5	2\\3	$\sqrt{15}$		
4. $18^{1/3}$ $\sqrt[3]{18}$	5. $32^{1/2}$ $4\sqrt{2}$	$6. \qquad \begin{pmatrix} \frac{2}{3} \\ \frac{81}{16} \end{pmatrix}$		
7. $\left(\frac{1}{10}\right)^{-2}$ 100	8. $\left(\frac{5}{2}\right)^{-3}$ $\frac{8}{125}$	9. $\left(\frac{6}{2}\right)^{-4}$ $\frac{1}{81}$		
	Proficient			
$10.\ 18^{2/3} \\ 3\sqrt[3]{2^2 \times 3} = 3\sqrt[3]{12}$	11. $32^{4/3}$ $2^{6}\sqrt[3]{2^2} = 64\sqrt[3]{4}$	12. $50^{3/2}$ 2 × 5 ³ √2 = 250√2		
13. $125^{2/3}$ $5^2 = 25$	14. $27^{5/3}$ $3^5 = 243$	15. 0.5^{-2} $2^2 = 4$		
16. $\left(\frac{1}{4}\right)^{-1/2}$	17. $(-1000)^{-2/3}$ $\frac{1}{100}$	18. $(-0.008)^{-4/_3}$ $5^4 = 625$		

Developing				
Express each radical as a power.				
1. $\sqrt{17^3}$	2. $\sqrt[4]{15^5}$	3. $\sqrt[2]{12^6}$		
17 ³ / ₂	15 ^{5/} 4	12 ²		
Proficient				
4. $\sqrt{(4x^2)^3}$	5. $\sqrt[3]{64x^6}$	6. $\left(\sqrt[4]{81a^8}\right)^2$		
$8x^3$	$4x^2$	$9a^4$		

Proficient							
Simplify	each expression w	ithout a	a calculator. Your answ	wers sh	ould contain only p	ositiv	e, whole number
exponer	nts.						
1.	$0.5x^{-2}$ $\frac{1}{2x^2}$	2.	$\left(\frac{a^2}{9}\right)^{-1/2}$ $\frac{3}{a}$	3.	$\frac{(-8q^6)^{-2/_3}}{\frac{1}{4q^4}}$	4.	$\frac{(-0.027m^3)^{-4/_3}}{\frac{10\ 000}{81m^4}}$
			Extend	ing			
5.	$\frac{\left(12x^3y^2\right)^{1/2}}{2xy\sqrt{3x}}$	6.	$(18a^2b^5)^{2/_3}$ $3ab^3\sqrt[3]{12ab}$	7.	$(32m^2n^8)^{3/_4} \\ 8mn^6 \sqrt[4]{8m^2}$	8.	
9.	$(64p^5q^9)^{4/_3}$ $2^8p^6q^{12}\sqrt[3]{p^2}$	10.	$3xy\left(\frac{x^2}{y^2}\right)^{-1/2}$ $3y^2$	11.	$\frac{(9a^3b^6)^{-1/_2}}{(3a^3b^6)^{-2}}\\3a^4b^9\sqrt{a}$	12.	$\frac{(8x^{n+2}y^{n+1})^{2/n}}{x^2y^2\sqrt[n]{2^6x^4y^2}}$

Proficient	Extending
Write each power as a radical in two different ways .	Write each power as a radical in two different ways,
	then write in simplest form without a calculator .
1. $26^{2/5}$	2. $20^{2/3}$
$\sqrt[5]{26^2} = (\sqrt[5]{26})^2$	$\sqrt[3]{20^2} = \left(\sqrt[3]{20}\right)^2 = 2\sqrt[3]{50}$
3. $19^{3/4}$	4. $25^{2/3}$
$\sqrt[4]{19^3} = (\sqrt[4]{19})^3$	$\sqrt[3]{25^2} = (\sqrt[3]{25})^2 = 5\sqrt[3]{5}$
5. $6^{5/2}$	6. $(9n^4)^{3/2}$
$\sqrt{6^5} = \left(\sqrt{6}\right)^5$	$\sqrt{(9n^4)^3} = \left(\sqrt{9n^4}\right)^3 = 27n^6$
7. $40^{2/3}$	8. $(40a^3b^4)^{2/3}$
$\sqrt[3]{40^2} = \left(\sqrt[3]{40}\right)^2$	$\sqrt[3]{(40a^3b^4)^2} = \left(\sqrt[3]{40a^3b^4}\right)^2 = 4a^2b^2\sqrt[3]{25b^2}$
9. $3^{5/4}$	10. $(72p^3q)^{5/2}$
$\sqrt[4]{3^5} = \left(\sqrt[4]{3}\right)^5$	$\sqrt{(72p^3q)^5} = \left(\sqrt{72p^3q}\right)^5 = 2^7 3^5 p^7 q^2 \sqrt{2pq}$

Extending
Arrange these numbers in order from least to greatest, without using a calculator.
$\sqrt[3]{9}, 9^{3/2}, 9, (\sqrt[5]{9})^4, 9^{1/2}$
$\sqrt[3]{9},9^{1/2}, (\sqrt[5]{9})^4,9,9^{3/2}$
Suppose you want \$5000 in three years. The interest rate for a savings account is 2.9% compounded
annually. The money, <i>P</i> dollars, you must invest now is given by the formula
$P = 5000(1.029)^{-3}$
How much must you invest now to have \$5000 in three years?
\$4589.06

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Learning Goal 4.4	Use exponent laws to simplify expressions with rational
	exponents.

Proficient		Extending		
Simplify without	using a calculator. Y	our answers sho	ould contain only positive, whol	e number exponents.
1.	$2m^2 \cdot 2m^3 = 4m^5$		2. $(x^{-2}x) = \frac{1}{x}$	⁻³) ⁴ 1 20
3.	$m^4 \cdot 2m^{-3} = 2m$		4. $(n^3)^{-1} = \frac{2}{n^3}$	$\frac{1}{2} \cdot 2n^{-1}$
5.	$\frac{a^3}{a^5} \cdot a^{-2}$ $= \frac{1}{a^4}$		$\begin{array}{l} 6. \qquad \qquad x^2 y^{-4} \\ = \frac{x}{y} \end{array}$	$(xy)^2$
7.	$4n^4 \cdot 2n^{-3} = 8n$		8. $(2x^4y^{-1}) = \frac{y}{2x^4}$	$(x^{-3})^{-1}$
9.	$\frac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4} = \frac{2x^2}{3yz^7}$		10. $\frac{(16a^2b)}{2ab} = \frac{16a^2b}{8a}$	$\frac{5}{b^3} - \frac{1}{2}$
11.	$2x^3y^{-3} \cdot 2x^{-1}y^3$ $= 4x^2$		12. $\left(\frac{x^2y}{y^{-2}}\right)$ $= \frac{1}{x^4}$	$\frac{1}{y^6}$
13.	$2y^{5/2} \cdot 3y^{-3}$ $= \frac{6\sqrt{y}}{y}$		14. $ba^4 \cdot (2b^6) = \frac{\sqrt{4a^6}}{4a^6}$	$(\frac{ba^4}{2})^{-3/2}$ $(\frac{ba^4}{2})^{-3/2}$
15.	$4v^{3} \cdot v^{-1/3}u^{-2} = \frac{4v^{2}\sqrt[3]{v^{2}}}{u}$		16. $\frac{(2x^{-3})}{x^3y^4z^2}$ $= \frac{8}{x^8}$	$\frac{z^2)^3}{x^{-4}z^3}$
17.	$4a^{3}b^{2} \cdot 3a^{-4}b^{-3} = \frac{12}{a^{4}b^{3}}$		18. $\frac{(16pm^{-1})^{-1/2}}{2pq} = \frac{p\sqrt[4]{2}}{2m}$	$\frac{4 \cdot 2m^{-1}p^3}{mp^3}$

19.	$\frac{2y^3 \cdot 3xy^3}{3x^2y^4} = \frac{2y^2}{x}$	20.	$(m \cdot m^{-2} \cdot n^{5/3})^2$ = $\frac{n^3 \sqrt[3]{n}}{m^2}$
21.	$4r^{-3} \cdot 2r^2 = \frac{8}{r}$	22.	$ \begin{pmatrix} \frac{x^{1/2}y^{-2}}{x^2y^{1/2}} \end{pmatrix}^4 \\ = \frac{1}{x^6y^{10}} $
23.	$\frac{3m^{-4}}{m^3} = \frac{3}{m^7}$	24.	$\frac{(x^3y^2)^{3/2}}{(x^{-1}y^{-2/3})^{1/4}}$ = $x^4y^3\sqrt[4]{x^3}\sqrt[6]{y}$ = $x^4y^{3\frac{12}{3}\sqrt{x^9y^2}}$
25.	$2k^4 \cdot 4k$ $= 8k^5$	26.	$\frac{(x^{-1/2}y^2)^{-5/4}}{x^2y^{1/2}} = \frac{\sqrt[8]{x^5}}{x^2y^3}$

Extending					
Write 3 different expressions for each result.					
1. $x^{3/2}$ as the product of two powers with rational exponents. $x \times x^{1/2} = x^{-1} \times x^{5/2} = x^{-3/2} \times x^3$					
2. $x^{3/2}$ as the quotient of two powers with rational exponents.					
$\frac{x^2}{x^{1/2}} = \frac{x}{x^{-1/2}} = \frac{x^{5/2}}{x}$					
3. $x^{3/2}$ as the result of raising a power with a rational exponent to a rational exponent.					
$\left(x^{1/2}\right)^3 = \left(x^3\right)^{1/2} = \left(x^{3/5}\right)^{5/2}$					
Simplify and write as both a power and a radical.					
$\binom{\sqrt[3]{x^4}}{\sqrt[5]{x^2}}$ $x^{\frac{26}{15}} = \binom{\sqrt[15]{x^2}}{\sqrt[5]{x^2}}$	$ \begin{pmatrix} \sqrt{y^3} \end{pmatrix} \begin{pmatrix} \sqrt{5} \sqrt{y^4} \end{pmatrix} $ $ x^{23/10} = \sqrt[10]{x^{23}} $	$\sqrt[3]{a^5b^6}$ $a^{5/6}b = b\sqrt[6]{a^5}$			