Name: _________________________ Date: ___________________

Learning Goal 0.2	Expectations for algebra from previous years.

More Questions - Solutions

Power Law	Product Law	Quotient Law	Change of Base
$\log_b x^y = y \log_b x$	$\log_b(xy) = \log_b x + \log_b y$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_b x = \frac{\log_a x}{\log_a b}$

1. Write each expression in terms of individual logarithms.

a. $\log_4 \frac{x}{yz}$

$$= \log_4 x - \log_4 yz$$
$$= \log_4 x - (\log_4 y + \log_4 z)$$
$$= \log_4 x - \log_4 y - \log_4 z$$

b. $\log_3\left(\frac{9}{\sqrt[3]{x^2}}\right)$

$$= \log_3 9 - \log_3 \sqrt[3]{x^2}$$
$$= 2 - \log_3 x^{2/3}$$
$$= 2 - \frac{2}{3}\log_3 x$$
$$= \frac{6 - 2\log_3 x}{3}$$
$$= \frac{2(3 - \log_3 x)}{3}$$

2. Evaluate using logarithm laws.

a. $\log_4 48 + \log_4\left(\frac{2}{3}\right) + \log_4 8$

$$= \log_4\left(48 \times \frac{2}{3}\right) + \log_4 8$$
$$= \log_4(32) + \log_4 8$$
$$= \log_4(32 \times 8)$$
$$= \log_4(256)$$
$$= \log_4(4^4)$$
$$= 4$$

b. $\log_6 \sqrt{12} + \log_6 \sqrt{3}$

$$= \log_6 \sqrt{12} \times \sqrt{3}$$
$$= \log_6 \sqrt{36}$$
$$= \log_6 6$$
$$= 1$$

c. $\log_{36} 2 - \log_{36} 12$

$$= \log_{36}\left(\frac{2}{12}\right)$$
$$= \log_{36}\left(\frac{1}{6}\right)$$
$$= -\frac{1}{2}$$

d. $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

$$= \log_3 6^2 - \log_3 \sqrt{64} + \log_3 2$$
$$= \log_3 36 - \log_3 8 + \log_3 2$$
$$= \log_3\left(\frac{36}{8}\right) + \log_3 2$$
$$= \log_3\left(\frac{9}{2}\right) + \log_3 2$$
$$= \log_3\left(\frac{9}{2} \times 2\right)$$
$$= \log_3(9)$$
$$= 2$$

3. Write as a single logarithm.

a. $\dfrac{n\log_a x}{\log_a y}$

$$= \frac{\log_a x^n}{\log_a y}$$
$$= \log_y x^n$$

b. $\dfrac{\log_6 64}{\log_6 4}$

$$= \log_4 64$$
$$= 3$$

c. $n\log_b x + \log_b x^{4-n} - \log_b x^{2n+3}$

$$= \log_b x^n + \log_b x^{4-n} - \log_b x^{2n+3}$$
$$= \log_b(x^n \times x^{4-n}) - \log_b x^{2n+3}$$
$$= \log_b\left(x^{n+(4-n)}\right) - \log_b x^{2n+3}$$
$$= \log_b x^4 - \log_b x^{2n+3}$$
$$= \log_b\left(\frac{x^4}{x^{2n+3}}\right)$$
$$= \log_b\left(x^{4-(2n+3)}\right)$$
$$= \log_b x^{1-2n}$$
$$= (1-2n)\log_b x$$

d. $\log_2(x^2-9) - \log_2(x^2-x-6)$

$$= \log_2\left(\frac{x^2-9}{x^2-x-6}\right)$$
$$= \log_2\left(\frac{(x+3)(x-3)}{(x-3)(x+2)}\right)$$
$$= \log_2\left(\frac{x+3}{x+2}\right)$$

$$x \neq -2, 3$$

4. Solve for the exact value of x. State any restrictions on the variable and verify your answers.

a. $2^z = 2500$

$$\begin{aligned}\log(2^x) &= \log(2500)\\ x\log(2) &= \log(2500)\\ x &= \frac{\log(2500)}{\log(2)}\\ x &\in \mathbb{R}\end{aligned}$$

b. $5^{x-3} = 1700$

$$\begin{aligned}\log(5^{x-3}) &= \log(1700)\\ (x-3)\log(5) &= \log(1700)\\ x-3 &= \frac{\log(1700)}{\log(5)}\\ x &= \frac{\log(1700)}{\log(5)} + 3\\ x &\in \mathbb{R}\end{aligned}$$

c. $8(3^{2x}) = 568$

$$\begin{aligned}3^{2x} &= 71\\ \log(3^{2x}) &= \log(71)\\ 2x\log(3) &= \log(71)\\ 2x &= \frac{\log(71)}{\log(3)}\\ x &= \frac{\log(71)}{2\log(3)}\\ x &\in \mathbb{R}\end{aligned}$$

d. $6^{3x+1} = 8^{x+3}$

$$\begin{aligned}\log(6^{3x+1}) &= \log(8^{x+3})\\ (3x+1)\log(6) &= (x+3)\log(8)\\ 3x\log 6 + \log 6 &= x\log 8 + 3\log 8\\ 3x\log 6 + \log 6 &= x\log 8 + \log 512\\ 3x\log 6 - x\log 8 &= \log 512 - \log 6\\ x(3\log 6 - \log 8) &= \log\left(\frac{512}{6}\right)\\ x(\log 216 - \log 8) &= \log\left(\frac{256}{3}\right)\\ x\log\left(\frac{216}{8}\right) &= \log\left(\frac{256}{3}\right)\\ x\log(27) &= \log\left(\frac{256}{3}\right)\\ x &= \log\left(\tfrac{256}{3}\right)\Big/\log(27),\ x \in \mathbb{R}\end{aligned}$$

e. $4(7^{x+2}) = 9^{2x-3}$

$$\begin{aligned}\log\left(4(7^{x+2})\right) &= \log(9^{2x-3})\\ \log(4) + \log(7^{x+2}) &= \log(9^{2x-3})\\ \log(4) + (x+2)\log(7) &= (2x-3)\log(9)\\ \log(4) + x\log(7) + 2\log(7) &= 2x\log(9) - 3\log(9)\\ \log(4) + x\log(7) + \log(49) &= x\log(81) - \log(729)\\ x\log(7) - x\log(81) &= -\log(729) - \log(4) - \log(49)\\ x(\log 7 - \log 81) &= \log\left(\frac{1}{729}\right) + \log\left(\frac{1}{4}\right) + \log\left(\frac{1}{49}\right)\\ x\log\left(\frac{7}{81}\right) &= \log\left(\frac{1}{729}\times\frac{1}{4}\times\frac{1}{49}\right)\\ x\log\left(\frac{7}{81}\right) &= \log\left(\frac{1}{142\,884}\right)\\ x &= \log\left(\tfrac{1}{142\,884}\right)\Big/\log\left(\tfrac{7}{81}\right),\ x \in \mathbb{R}\end{aligned}$$

f. $\log_7 x + \log_7 4 = \log_7 12$

$$\log_7 x = \log_7 12 - \log_7 4$$
$$\log_7 x = \log_7\left(\frac{12}{4}\right)$$
$$\log_7 x = \log_7(3)$$
$$7^{\log_7 x} = 7^{\log_7(3)}$$
$$x = 3$$

NPVs: $x > 0$

$$3 > 0$$

$$\log_7(3) + \log_7 4 = \log_7 12$$
$$\log_7(3 \times 4) = \log_7 12$$
$$\log_7(12) = \log_7 12$$

g. $\log_2(x-6) = 3 - \log_2(x-4)$

$$\log_2(x-6) + \log_2(x-4) = 3$$
$$\log_2\big((x-6)(x-4)\big) = 3$$
$$\log_2(x^2 - 10x + 24) = 3$$
$$2^{\log_2(x^2-10x+24)} = 2^3$$
$$x^2 - 10x + 24 = 8$$
$$x^2 - 10x + 16 = 0$$
$$(x-2)(x-8) = 0$$
$$x = 2, 8$$

NPVs: $x - 6 > 0$, $x > 6$; $x - 4 > 0$, $x > 4$

$2 < 6$
Extraneous Root

$8 > 6$

$$\log_2\big((8)-6\big) = 3 - \log_2\big((8)-4\big)$$
$$\log_2(2) = 3 - \log_2(4)$$
$$1 = 3 - 2$$
$$1 = 1$$

h. $\log_3(x^2 - 8x)^5 = 10$

$$5\log_3(x^2 - 8x) = 10$$
$$\log_3(x^2 - 8x) = 2$$
$$3^{\log_3(x^2-8x)} = 3^2$$
$$x^2 - 8x = 9$$
$$x^2 - 8x - 9 = 0$$
$$(x-9)(x+1) = 0$$
$$x = -1, 9$$

NPVs: $x^2 - 8x > 0$

$$x(x-8) > 0$$

$x > 0$; $x > 8$

$-1 < 8$
Extraneous Root

$9 > 8$

$$\log_3\big((9)^2 - 8(9)\big)^5 = 10$$
$$\log_3(81 - 72)^5 = 10$$
$$\log_3(9)^5 = 10$$
$$\log_3(3)^{10} = 10$$
$$10 = 10$$

i. $\log_2(x+3)^2 = 4$

NPVs: $x + 3 > 0$, $x > -3$

$$2\log_2(x+3) = 4$$
$$\log_2(x+3) = 2$$
$$2^{\log_2(x+3)} = 2^2$$
$$x + 3 = 4$$
$$x = 1$$

$$1 > -3$$

$$\log_2\big((1)+3\big)^2 = 4$$
$$\log_2(4)^2 = 4$$
$$\log_2(2)^4 = 4$$
$$4 = 4$$