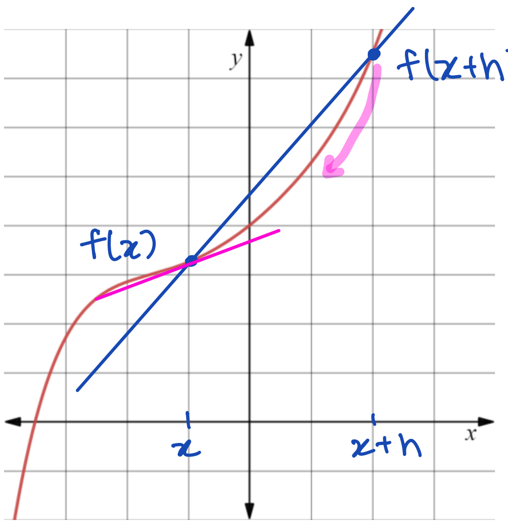
Name: ___________________________ Date: ____________________

Learning Goal 2.2	Limits at infinity and the definition of the derivative

Recall the difference quotient

slope btwn a fixed pt & a movable pt. $= \dfrac{f(x+h)-f(x)}{h}$

Definition of the derivative



Derivative of $f(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$\frac{dy}{dx}$

Derivative of $f(a)$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

Example Find the equation of the tangent line to the parabola $y = x^2$ at the point $P(1,1)$.

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{x^2} + 2xh + h^2) - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 2x+h$$

$$= 2x$$

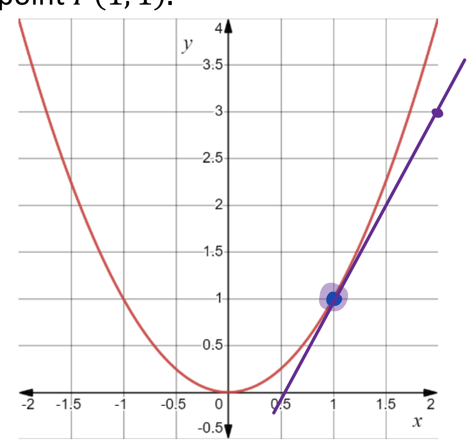
$f'(1) = 2(1)$
$= 2$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x-1)$

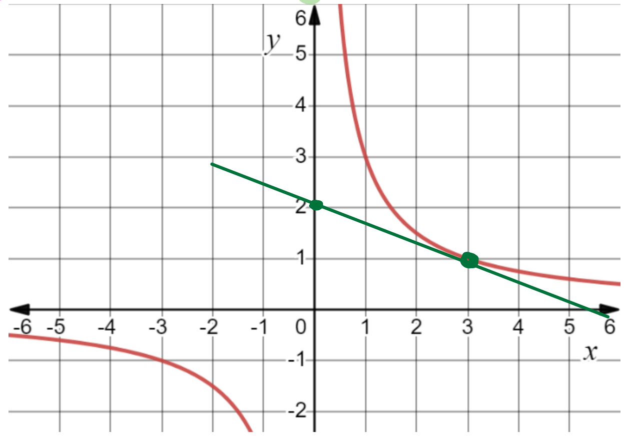
$y - 1 = 2x - 2$

$y = 2x - 1$



Example What is the slope of the tangent line to the function $y = {}^{3}/_{x}$ at the point $P(3, 1)$.

take the derivative!



$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\left(\frac{3}{x+h}\right) - \left(\frac{3}{x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3x - 3(x+h)}{x(x+h)}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \frac{\cancel{3x} - \cancel{3x} - 3h}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-3\cancel{h}}{\cancel{h}x(x+h)}$$

$$= \lim_{h\to 0} \frac{-3}{x(x+h)}$$

$$= -\frac{3}{x^2}$$

$$\frac{dy}{dx} \text{ at } (3,1) = -\frac{3}{(3)^2} = -\frac{3}{9} = -\frac{1}{3}$$

Example Find $R'(x)$ given $R(x) = \sqrt{5x - 8}$ using the definition of the derivative.

no need to evaluate the derivative anywhere.

$$R'(x) = \lim_{h\to 0} \frac{\sqrt{5(x+h)-8} - \sqrt{5x-8}}{h} \times \frac{\sqrt{5(x+h)-8} + \sqrt{5x-8}}{\sqrt{5(x+h)-8} + \sqrt{5x-8}}$$

$$= \lim_{h\to 0} \frac{(5(x+h)-8) - (5x-8)}{\underbrace{h\left(\sqrt{5(x+h)-8} + \sqrt{5x-8}\right)}_{y}}$$

$$= \lim_{h\to 0} \frac{\cancel{5x} + 5h - \cancel{8} - \cancel{5x} + \cancel{8}}{y}$$

$$= \lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}\left(\sqrt{5(x+\cancel{h})-8} + \sqrt{5x-8}\right)}$$

when we take the limit as $h \to 0$

$$R'(x) = \frac{5}{2\sqrt{5x-8}}$$

Assignment # 1 – 9, 13 – 21, 24 – 32 Quiz Next Day!