Name: _________________________ Date: ___________________

Learning Goal 2.2	Limits at infinity and the definition of the derivative

More Questions – Solutions

1. Find the equation of the tangent line to the given functions using the definition of the derivative.

a. $f(x) = x^2 \qquad x = -3$

$$f'(x) = \lim_{h\to o} \frac{(x+h)^2 - x^2}{h}$$
$$= \lim_{h\to o} \frac{x^2 + 2xh + h^2 - x^2}{h}$$
$$= \lim_{h\to o} \frac{2xh + h^2}{h}$$
$$= \lim_{h\to o} \frac{h(2x+h)}{h}$$
$$= \lim_{h\to o} (2x+h)$$
$$= 2x$$

$$f'(-3) = 2(-3) = -6 \qquad f(-3) = (-3)^2 = 9$$

$$y - 9 = -6(x+3)$$
$$y - 9 = -6x - 18$$
$$y = -6x - 9$$

b. $h(x) = x^3 - 4 \qquad x = 1$

$$h'(1) = \lim_{x\to 1} \frac{(x^3-4) - (1-4)}{x-1}$$
$$= \lim_{x\to 1} \frac{(x^3-4)+3}{x-1}$$
$$= \lim_{x\to 1} \frac{x^3-1}{x-1}$$
$$= \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1}$$
$$= \lim_{x\to 1} x^2 + x + 1$$
$$= (1)^2 + (1) + 1$$
$$= 3$$

$$h(1) = -3$$

$$y + 3 = 3(x-1)$$
$$y + 3 = 3x - 3$$
$$y = 3x - 6$$

c. $g(x) = \dfrac{x}{x+1} \qquad x = -2$

$$g'(-2) = \lim_{x\to -2} \frac{\frac{x}{x+1} - \frac{(-2)}{(-2)+1}}{x+2}$$
$$= \lim_{x\to -2} \frac{\frac{x}{x+1} - 2}{x+2}$$
$$= \lim_{x\to -2} \frac{\frac{x - 2(x+1)}{x+1}}{x+2}$$
$$= \lim_{x\to -2} \frac{x - 2(x+1)}{(x+1)(x+2)}$$
$$= \lim_{x\to -2} \frac{x - 2x - 2}{(x+1)(x+2)}$$
$$= \lim_{x\to -2} \frac{-x-2}{(x+1)(x+2)}$$
$$= \lim_{x\to -2} \frac{-1}{x+1}$$
$$= \frac{-1}{(-2)+1}$$
$$= 1$$

$$g(-2) = 2$$

$$y - 2 = 1(x+2)$$
$$y - 2 = x + 2$$
$$y = x + 4$$

2. What is the slope of the tangent line to the functions, at the given point, using the definition of the derivative.

a. $f(x) = \sqrt{x}$ $(4, 2)$

$$f'(x) = \lim_{h \to o} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$
$$= \lim_{h \to o} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$
$$= \lim_{h \to o} \frac{(x+h) - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$
$$= \lim_{h \to o} \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$
$$= \lim_{h \to o} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

b. $g(x) = \frac{1}{x-2}$ $(3, 1)$

$$g'(3) = \lim_{x \to 3} \frac{\frac{1}{x-2} - \frac{1}{3-2}}{x-3}$$
$$= \lim_{x \to 3} \frac{\frac{1}{x-2} - 1}{x-3}$$
$$= \lim_{x \to 3} \frac{\frac{1-(x-2)}{x-2}}{x-3}$$
$$= \lim_{x \to 3} \frac{\frac{3-x}{x-2}}{x-3}$$
$$= \lim_{x \to 3} \frac{3-x}{(x-3)(x-2)}$$
$$= \lim_{x \to 3} \frac{-1}{(x-2)}$$
$$= \frac{-1}{(3-2)}$$
$$= -1$$

c. $h(x) = 5x^2 - 4$ $(2, 16)$

$$\lim_{h \to 0} \frac{5(x+h)^2 - 4 - (5x^2 - 4)}{h}$$
$$= \lim_{h \to 0} \frac{5(x^2 + 2xh + h^2) - 4 - 5x^2 + 4}{h}$$
$$= \lim_{h \to 0} \frac{5x^2 + 10xh + 2h^2 - 5x^2}{h}$$
$$= \lim_{h \to 0} \frac{10xh + 2h^2}{h}$$
$$= \lim_{h \to 0} \frac{2h(5x+h)}{h}$$
$$= \lim_{h \to 0} 2(5x+h)$$
$$= 10x$$

$$h'(2) = 10(2) = 20$$

3. Find the derivative using the definition of the derivative.

a. $h(x) = \frac{1}{x^2}$

$$h'(x) = \lim_{h \to o} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$
$$= \lim_{h \to o} \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h}$$
$$= \lim_{h \to o} \frac{x^2 - (x+h)^2}{hx^2(x+h)^2}$$
$$= \lim_{h \to o} \frac{x^2 - x^2 - 2xh - h^2}{hx^2(x+h)^2}$$
$$= \lim_{h \to o} \frac{-2xh - h^2}{hx^2(x+h)^2}$$
$$= \lim_{h \to o} \frac{-h(2x+h)}{hx^2(x+h)^2}$$
$$= \lim_{h \to o} \frac{-(2x+h)}{x^2(x+h)^2}$$
$$= \frac{-2x}{x^4}$$
$$= \frac{-2}{x^3}$$

b. $g(x) = 8x - 1$

$$= \lim_{h \to o} \frac{8(x+h) - 1 - (8x - 1)}{h}$$
$$= \lim_{h \to o} \frac{8x + 8h - 1 - 8x + 1}{h}$$
$$= \lim_{h \to o} \frac{8h}{h}$$
$$= \lim_{h \to o} 8$$
$$= 8$$

c. $f(x) = 7x^2$

$$f'(x) = \lim_{h \to o} \frac{7(x+h)^2 - 7x^2}{h}$$
$$= \lim_{h \to o} \frac{7(x^2 + 2xh + h^2) - 7x^2}{h}$$
$$= \lim_{h \to o} \frac{7x^2 + 14xh + 7h^2 - 7x^2}{h}$$
$$= \lim_{h \to o} \frac{14xh + 7h^2}{h}$$
$$= \lim_{h \to o} \frac{7h(2x+h)}{h}$$
$$= \lim_{h \to o} 7(2x+h)$$
$$= 14x$$

Assignment # 1 – 9, 13 – 21, 24 – 32 Quiz Next Day!