

Name: \_\_\_\_\_

Date: \_\_\_\_\_

<b>Learning Goal 5.4</b>	Solve radical equations, identifying extraneous roots and restrictions to the domain.
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**Recall** Order of Operations. Use it to solve the following linear equation.

↑ Brackets.  
E  
D  
M  
A  
S

$$\begin{aligned}
 5 + 2(2x - 1) &= 13 \\
 -5 & \qquad \qquad -6 \\
 \frac{2(2x - 1)}{2} &= \frac{8}{2} \\
 2x - 1 &= 4 \\
 +1 & \qquad +1 \\
 \frac{2x}{2} &= \frac{5}{2} \\
 x &= \frac{5}{2}
 \end{aligned}$$

**Example** State any restrictions on the variable, if any. Solve.

a.  $5 + \sqrt{2x - 1} = 12$

b.  $\sqrt{2x - 1} + 5 = 2$

NPVS:  $2x - 1 \geq 0$        $x \geq \frac{1}{2}$

$$\begin{aligned}
 &+1 \quad +1 \\
 \frac{2x}{2} &\geq \frac{1}{2}
 \end{aligned}$$

NPVS:  $2x - 1 \geq 0$        $x \geq \frac{1}{2}$

$$\begin{aligned}
 5 + \sqrt{2x - 1} &= 12 \\
 -5 & \qquad \qquad -5 \\
 (\sqrt{2x - 1})^2 &= (7)^2 \\
 2x - 1 &= 49 \\
 +1 & \qquad +1 \\
 \frac{2x}{2} &= \frac{50}{2} \\
 x &= 25 \geq \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{2x - 1} + 5 &= 2 \\
 -5 & \qquad -5 \\
 (\sqrt{2x - 1})^2 &= (-3)^2 \\
 2x - 1 &= 9 \\
 +1 & \qquad +1 \\
 \frac{2x}{2} &= \frac{10}{2} \\
 x &= 5 \geq \frac{1}{2}
 \end{aligned}$$

CHECK

$$\begin{aligned}
 5 + \sqrt{2(25) - 1} &\stackrel{?}{=} 12 \\
 &= 5 + \sqrt{49} \\
 &= 5 + 7 \\
 &= 12 \checkmark
 \end{aligned}$$

CHECK

$$\begin{aligned}
 \sqrt{2(5) - 1} + 5 &\stackrel{?}{=} 2 \\
 &= \sqrt{9} + 5 \\
 &= 3 + 5 \\
 &= 8 \neq 2
 \end{aligned}$$

extraneous

No real solutions.

$$\begin{aligned}
 \text{c. } & -8 + \sqrt{\frac{3y}{5}} = -2 \\
 & +8 \qquad \qquad +8 \\
 & \left(\sqrt{\frac{3y}{5}}\right)^2 = (6)^2 \\
 & 5 \times \frac{3y}{5} = 36 \times 5 \\
 & 3y = 180 \\
 & \frac{3y}{3} = \frac{180}{3} \\
 & y = 60 \geq 0
 \end{aligned}$$

CHECK

$$\begin{aligned}
 & -8 + \sqrt{\frac{3(60)}{5}} \stackrel{?}{=} -2 \\
 & = -8 + \sqrt{36} \\
 & = -8 + 6 \\
 & = -2 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \text{NPVS:} \\
 \frac{3y}{5} & \geq 0 \\
 y & \geq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } & \sqrt[3]{3x-1} + 7 = 3 \\
 & \text{NPVS: none.} \\
 & \sqrt[3]{3x-1} + 7 = 3 \\
 & \qquad \qquad -7 \quad -7 \\
 & \left(\sqrt[3]{3x-1}\right)^3 = (-4)^3 \\
 & 3x-1 = -64 \\
 & \qquad \qquad +1 \quad +1 \\
 & 3x = -63 \\
 & \frac{3x}{3} = \frac{-63}{3} \\
 & x = -21
 \end{aligned}$$

CHECK:

$$\begin{aligned}
 & \sqrt[3]{3(-21)-1} + 7 \stackrel{?}{=} 3 \\
 & = \sqrt[3]{-64} + 7 \\
 & = -4 + 7 \\
 & = 3 \checkmark
 \end{aligned}$$

**Example** An observer is in a hot air balloon that is attached to the top of a 200 metre tower whose base is at sea level. How high above the tower must the balloon be so the observer's distance to the horizon is 100 km?