Name: _____ Date: _____

Learning Goal 5.4	Solve radical equations, identifying extraneous roots and restrictions to the domain.

Example State any restrictions on the variable, if any. Solve.

a. $n - \sqrt{5-n} = -7$

$-n \quad -n$

$-\sqrt{5-n} = -7-n \quad \times -1$

$(\sqrt{5-n})^2 = (7+n)^2$

$5-n = (7+n)(7+n)$

$5-n = 49+14n+n^2$

$-5+n \quad -5 \quad +n$

$0 = 44+15n+n^2$

$11 \times 4 = 44$

$11 + 4 = 15$

$0 = (n+11)(n+4)$

$n+11=0 \qquad n+4=0$

$-11 \quad -11 \qquad -4 \quad -4$

$n = -11 \leq 5 \qquad n = -4 \leq 5$

extraneous root

CHECK

$(-11) - \sqrt{5-(-11)} \stackrel{?}{=} -7$

$= -11 - \sqrt{16}$

$= -11 - 4$

$= -15$

$(-4) - \sqrt{5-(-4)} \stackrel{?}{=} -7$

$= -4 - \sqrt{9}$

$= -4 - 3$

$= -7$

NPVS:

$5-n \geq 0$

$-5 \quad -5$

$-n \geq -5$

$-1 \quad -1$

$n \leq 5$

b. $(\sqrt{y-3})^2 = (y-3)^2$

NPVS:

$y-3 \geq 0$

$y \geq 3$

$y-3 = (y-3)(y-3)$

$y-3 = y^2-6y+9$

$-y+3 \quad -y+3$

$0 = y^2-7y+12$

$-3 \times -4 = 12$

$-3 + -4 = -7$

$0 = (y-3)(y-4)$

$y-3=0 \qquad y-4=0$

$y=3 \geq 3 \qquad y=4 \geq 3$

CHECK:

$\sqrt{(3)-3} = (3)-3$

$= 0 \qquad = 0$

$\sqrt{(4)-3} = (4)-3$

$= \sqrt{1} \qquad = 1$

$= 1$

c. $\sqrt{8-\frac{m}{3}} = \sqrt{3m} - 4$

NPVS: $8 - \frac{m}{3} \geq 0$ $\quad m \geq 0$

$8 \geq \frac{m}{3}$

$24 \geq m$

$0 \leq m \leq 24$

$\left(\sqrt{8-\frac{m}{3}}\right)^2 = \left(\sqrt{3m} - 4\right)^2$

$8 - \frac{m}{3} = (\sqrt{3m} - 4)(\sqrt{3m} - 4)$

$8 - \frac{m}{3} = 3m - 8\sqrt{3m} + 16$

$-16 \quad -3m \quad -16$

$-8 - \frac{10m}{3} = -8\sqrt{3m}$ $\quad \times -1$

$\left(8 + \frac{10m}{3}\right)^2 = \left(8\sqrt{3m}\right)^2$

$64 + \frac{160m}{3} + \frac{100m^2}{9} = 64(3m)$

$64 + \frac{160}{3}m + \frac{100}{9}m^2 = 192m$ $\quad \times 9$

$576 + 480m + 100m^2 = 1728m$

$-1728 \quad -1728m$

$100m^2 - 1248m + 576 = 0$

$a \quad b \quad c$

$m = \frac{1248 \pm \sqrt{(-1248)^2 - 4(100)(576)}}{2(100)}$

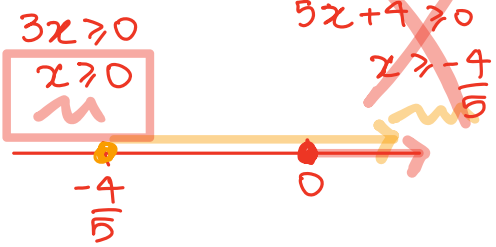
$= \frac{1248 \pm \sqrt{1327104}}{200}$

$= \frac{1248 \pm 1152}{200}$

$0 \leq m_+ = 12 \leq 24$ $\quad 0 \leq m_- = \frac{12}{25} \leq 24$ ← extraneous root

d. $7 + \sqrt{3x} = \sqrt{5x+4} + 5$

NPVS: $3x \geq 0$ $\quad x \geq 0$ $\quad 5x+4 \geq 0$ $\quad x \geq -\frac{4}{5}$



$7 + \sqrt{3x} = \sqrt{5x+4} + 5$

$-5 \quad -5$

$\left(2 + \sqrt{3x}\right)^2 = \left(\sqrt{5x+4}\right)^2$

$(2+\sqrt{3x})(2+\sqrt{3x}) = 5x+4$

$4 + 4\sqrt{3x} + 3x = 5x+4$

$-4 \quad -3x \quad -3x-4$

$\left(4\sqrt{3x}\right)^2 = (2x)^2$

$16(3x) = 4x^2$

$48x = 4x^2$

$-48x \quad -48x$

$0 = 4x^2 - 48x$

$= 4x(x-12)$

$x = 0 \geq 0$ $\quad x = 12 \geq 0$

CHECK

$7 + \sqrt{3(0)} = \sqrt{5(0)+4} + 5$

$= 7$ $\quad = \sqrt{4} + 5$

$= 2 + 5$

$= 7$ ✓

$7 + \sqrt{3(12)} = \sqrt{5(12)+4} + 5$

$= 7 + \sqrt{36}$ $\quad = \sqrt{64} + 5$

$= 13$ ✓ $\quad = 13$

CHECK

$$\sqrt{8-\frac{12}{3}} = \sqrt{3(12)} - 4$$

$$= \sqrt{8-4} \qquad = \sqrt{36} - 4$$

$$= \sqrt{4} \qquad = 6 - 4$$

$$= 2 \qquad = 2 \quad ✓$$

$$\sqrt{8-\frac{\left(\frac{12}{25}\right)}{3}} = \sqrt{3\left(\frac{12}{25}\right)} - 4$$

$$= \sqrt{8-\frac{12}{75}} \qquad = \sqrt{\frac{36}{25}} - 4$$

$$= \sqrt{\frac{588}{75}} \qquad = \frac{6}{5} - 4$$

$$= \frac{14}{5} \quad ✗ \qquad = -\frac{14}{5}$$