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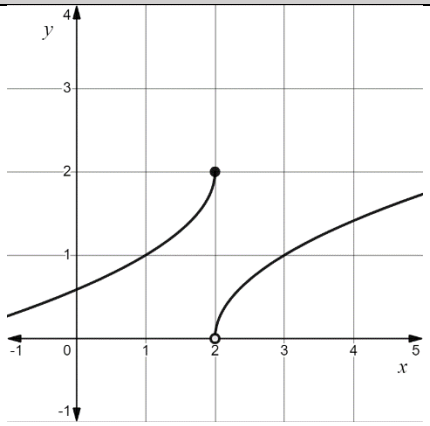
Chapter 2 Review
Limits and Derivatives

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 2.1	Finite limits and continuity.

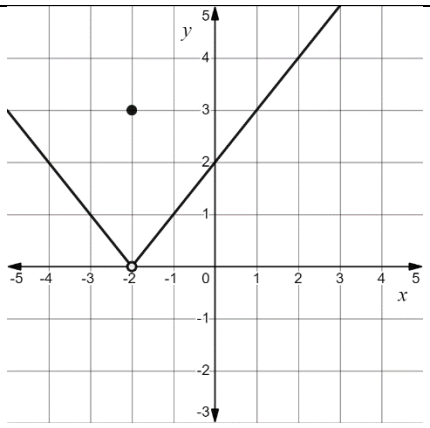
1. Find the following limits.

Developing



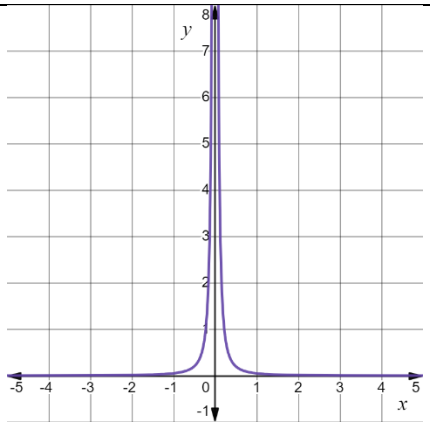
a. $\lim_{x\to 2^-} f(x)$ b. $\lim_{x\to 2^+} f(x)$

c. $\lim_{x\to 2} f(x)$



d. $\lim_{x\to -2^-} f(x)$ e. $\lim_{x\to -2^+} f(x)$

f. $\lim_{x\to -2} f(x)$



g. $\lim_{x\to 0^-} f(x)$ h. $\lim_{x\to 0^+} f(x)$

i. $\lim_{x\to 0} f(x)$

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j. $\lim_{x\to -2} x^3+6x^2-16$	k. $\lim_{x\to 4}\frac{x^2+9}{x^2-1}$	l. $\lim_{x\to 4}\frac{x^2-16}{x^2+x-20}$
m. $\lim_{x\to 0}\frac{x^2+2x}{x-2x^2}$	n. $\lim_{x\to 1}\frac{1-x^2}{x^2+5x-6}$	o. $\lim_{x\to 1}\frac{x^2+x-2}{x^2-4x+3}$
p. $\lim_{x\to 3}\frac{x^3-27}{x-3}$	q. $\lim_{x\to 1}\frac{x^3-3x^2+2x}{x-1}$	r. $\lim_{x\to 2}\frac{x^3-4x}{x^3-2x^2}$
s. $\lim_{x\to a}\frac{1/x-1/a}{x-a}$	t. $\lim_{x\to 0}\frac{1/(3+x)-1/3}{x}$	u. $\lim_{x\to -a}\frac{x^3+a}{x+a}$
v. $\lim_{x\to 3}\frac{x-3}{x^3-27}$	w. $\lim_{x\to 2}\frac{1-4/x^2}{1-2/x}$	x. $\lim_{x\to 4^-}\frac{x-4}{\lvert x-4\rvert}$
y. $\lim_{x\to 1}\frac{x-1}{\lvert x-1\rvert}$	z. $\lim_{x\to 1}\begin{cases}\frac{1}{x+2}, & x<1\\ 1-2x, & x>1\end{cases}$	aa. $\lim_{x\to 3}\begin{cases}x^2-1, & x<3\\ (x-1)^3, & x>3\end{cases}$
bb. $\lim_{x\to 3}\frac{4x^2-36}{2x-6}$	cc. $\lim_{x\to -1}\frac{x^3+1}{x^4-1}$	dd. $\lim_{x\to 2}\frac{2x^2-x-6}{3x^2-7x+2}$

Proficient

a. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$	b. $\lim_{x\to 25}\frac{5-\sqrt{x}}{25-x}$	c. $\lim_{x\to 9}\frac{9-x}{\sqrt{x}-3}$
d. $\lim_{x\to 0}\frac{(x+3)^3-27}{x}$	e. $\lim_{x\to 0}\frac{x^2}{\sqrt{x^2+12}-\sqrt{12}}$	f. $\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{6}{x^2-9}\right)$
g. $\lim_{x\to 5}\frac{x-5}{\sqrt{x-1}-2}$	h. $\lim_{x\to 2}\frac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x}$	i. $\lim_{x\to 16}\frac{4-\sqrt{x}}{x-16}$
j. $\lim_{x\to 2}\frac{x^2-x-2}{\lvert x-2\rvert}$	k. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$	l. $\lim_{x\to 0}\frac{2-\sqrt{4+x}}{x}$

Extending

a. $\lim_{x\to 0}\frac{\sin 2x}{4x}$	b. $\lim_{x\to 0}\frac{\sin x}{x^2-3x}$	c. $\lim_{x\to 0}\frac{\sin x+3x+1}{x}$
d. $\lim_{x\to 0}\frac{x\sin x}{\lvert x\rvert}$	e. $\lim_{x\to 0}\frac{(x+8)^{1/3}-2}{x}$	f. $\lim_{x\to 0}\frac{\sqrt{x+1}-\sqrt{2x+1}}{\sqrt{3x+4}-\sqrt{2x+4}}$
g. $\lim_{x\to 1}\frac{x^{1/6}-1}{x-1}$	h. $\lim_{x\to 5/2}\frac{\lvert 2x-5\rvert(x+1)}{2x-5}$	i. $\lim_{x\to 1}\frac{x^2+\lvert x-1\rvert-1}{\lvert x-1\rvert}$
j. $\lim_{x\to 27}\frac{27-x}{x^{1/3}-3}$	k. $\lim_{x\to 4}\frac{\sqrt{x}-2}{\sqrt{x^3}-8}$	l. $\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}$
m. $\lim_{x\to 1^+}\frac{1/x-1}{x^2-2x+1}$	n. $\lim_{x\to 0^+}\frac{3+x^{-1/2}+x^{-1}}{2+4x^{-1/2}}$	o. $\lim_{x\to 0^+}(x+5)\left(\frac{1}{2x}+\frac{1}{x+2}\right)$
p. $\lim_{x\to 2}\frac{x^3-6x-2}{x^3-4x}$		

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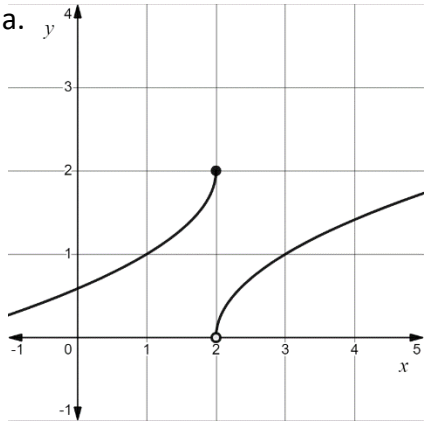
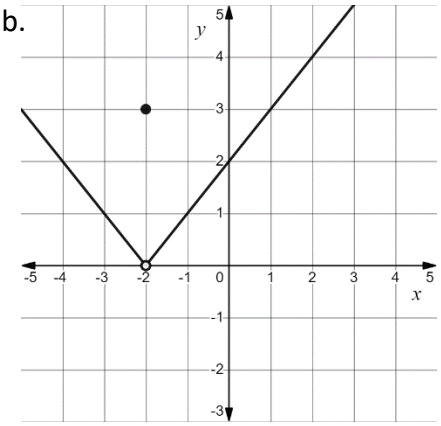
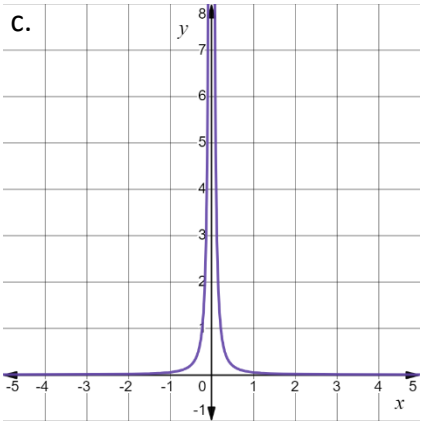
2. Find the point(s) and the associated type(s) of discontinuity.

Developing

a. 

b. 

c. 

d. $f(x) = \dfrac{x-1}{x^2+2x-8}$

e. $f(x) = \dfrac{x^2-16}{x^2+x-20}$

f. $f(x) = \dfrac{x^2+2x}{x-2x^2}$

g. $f(x) = \dfrac{1-x^2}{x^2+5x-6}$

h. $f(x) = \dfrac{x^2+x-2}{x^2-4x+3}$

i. $f(x) = \dfrac{x^3-4x}{x^3-2x^2}$

j. $f(x) = \dfrac{2x^2+5x+20}{x^2+4x}$

k. $f(x) = \dfrac{x^3+1}{x^4-1}$

l. $f(x) = \dfrac{2x^2-x-6}{3x^2-7x+2}$

Proficient

a. $f(x) = \dfrac{x-4}{|x-4|}$

b. $f(x) = \begin{cases} \dfrac{2}{x-1}, & x < 2 \\ x^3 - 2x + 1, & x \geq 2 \end{cases}$

c. $f(x) = \dfrac{9-x}{\sqrt{x}-3}$

Extending

a. $f(x) = \dfrac{x-3}{x^3-27}$

b. $f(x) = \dfrac{x^3-3x-10}{x^3-5x^2-4x+20}$

c. $f(x) = \dfrac{3x^3-5x^2-4x+4}{3x^3-8x^2+3z+2}$

3. Determine constants a and b such that $f(x)$ is continuous for all values of x.

$$f(x) = \begin{cases} ax+3, & x > 5 \\ 8, & x = 5 \\ x^2+bx+a, & x < 5 \end{cases}$$

$$a = 1, b = -\frac{18}{5}$$

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Limits and Derivatives

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Chapter 2 Review
Limits and Derivatives

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 2.2	Infinite limits and the definition of the derivative.

1. Determine the value of the infinite limit.

Developing		
a. $\lim_{x\to\infty} \frac{2x^2-3x+7}{x^2+47x+1}$	b. $\lim_{x\to\infty} \frac{x^2-16}{x^2+x-20}$	c. $\lim_{x\to\infty} \frac{2x^2+3}{5x^2+x}$
d. $\lim_{x\to\infty} \frac{1-x^2}{x^2+5x-6}$	e. $\lim_{x\to\infty} \frac{x^2+x-2}{3x^2-4x+3}$	f. $\lim_{x\to\infty} \frac{x^2-4x}{x^3-2x^2}$
g. $\lim_{x\to\infty} \frac{5x^3-3x^2+1}{x^2+2x+4}$	h. $\lim_{x\to-\infty} \frac{3x^3+x^2+1}{x^3+1}$	i. $\lim_{x\to\infty} \frac{x^5-x^3+x-1}{x^6+2x^2+1}$
j. $\lim_{x\to-\infty} (2x^3-x)$	k. $\lim_{x\to-\infty} \frac{x+2}{x^2+x+1}$	l. $\lim_{x\to-\infty} \frac{3x^3}{3x^2-2}$
m. $\lim_{x\to-\infty} \frac{2x^2}{x^2-4}$	n. $\lim_{x\to\infty} -\frac{3x^2}{4x+4}$	o. $\lim_{x\to\infty} \frac{2x^3}{3x^2-4}$
p. $\lim_{x\to-\infty} \frac{4x^3}{4x^2+3}$	q. $\lim_{x\to\infty} \frac{x+1}{2x^2+2x+1}$	r. $\lim_{x\to-\infty} \frac{\sqrt{2x^2+3}}{2x+3}$
s. $\lim_{x\to-\infty} \frac{\sqrt{2x^2+1}}{4x+2}$	t. $\lim_{x\to-\infty} \frac{4x+8}{5x}$	u. $\lim_{x\to-\infty} \frac{5x^2}{x+3}$
Proficient		
a. $\lim_{x\to\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}}$	b. $\lim_{x\to\infty} \frac{x+5-{}^2/_x-{}^1/_{x^3}}{3x+12-{}^1/_{x^2}}$	c. $\lim_{x\to\infty} \frac{x+x^{1/2}+x^{1/3}}{x^{2/3}+x^{1/4}}$
d. $\lim_{x\to\infty} \frac{1-({}^x/_{x-1})}{1-\sqrt{{}^x/_{x-1}}}$	e. $\lim_{x\to-\infty} \frac{x+x^{-1}}{1+\sqrt{1-x}}$	f. $\lim_{x\to\infty} \frac{x^{-1}+x^{-1/2}}{x+x^{-1/2}}$
g. $\lim_{x\to\infty} \frac{x+x^{-2}}{2x+x^{-2}}$	h. $\lim_{x\to\infty} \frac{5+x^{-1}}{1+2x^{-1}}$	i. $\lim_{x\to\infty} \frac{4x}{\sqrt{2x^2+1}}$
j. $\lim_{x\to\infty} (x+5)\left(\frac{1}{2x}+\frac{1}{x+2}\right)$	k. $\lim_{x\to-\infty} \frac{x^4+1}{x^3-1}$	l. $\lim_{x\to\infty} \left(\frac{\ln x}{x^4}+1\right)$
m. $\lim_{x\to\infty} (-e^{-3x}-1)$	n. $\lim_{x\to\infty} (e^x-3)$	o. $\lim_{x\to-\infty} -e^{-4x}$

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Extending		
a. $\lim_{x\to\infty}(\sqrt{x^2+x}-\sqrt{x^2-x})$	b. $\lim_{x\to\infty}\dfrac{1-\sqrt{x/x+1}}{2-\sqrt{4x+1/x+2}}$	c. $\lim_{x\to\infty}\dfrac{e^x+x^4}{x^3+5\ln x}$
If $1<a<b$ then $f(x)=b^x$ grows faster than $g(x)=a^x$ as $x\to\infty$. Use this idea for the following questions.		
d. $\lim_{x\to\infty}\dfrac{2^x+5(3^x)}{3(2^x)-3^x}$	e. $\lim_{x\to-\infty}\dfrac{2^x+5(3^x)}{3(2^x)-3^x}$	

2. Find the equation(s) of any and all asymptotes that exist (vertical, horizontal or slant).

Proficient		
a. $f(x)=\dfrac{2x^2-3x+7}{x^2+47x+1}$	b. $f(x)=\dfrac{x^2-16}{x^2+x-20}$	c. $f(x)=\dfrac{2x^2+3}{5x^2+x}$
d. $f(x)=\dfrac{1-x^2}{x^2+5x-6}$	e. $f(x)=\dfrac{x^2+x-2}{3x^2-4x+3}$	f. $f(x)=\dfrac{x^2-4x}{x^3-2x^2}$
g. $f(x)=\dfrac{5x^3-3x^2+1}{x^2+2x+4}$	h. $f(x)=\dfrac{3x^3+x^2+1}{x^3+1}$	i. $f(x)=\dfrac{x^5-x^3+x-1}{x^6+2x^2+1}$
j. $f(x)=\dfrac{x+2}{x^2+x+1}$	k. $f(x)=\dfrac{3x^3}{3x^2-2}$	l. $f(x)=\dfrac{2x^2}{x^2-4}$
m. $f(x)=-\dfrac{3x^2}{4x+4}$	n. $f(x)=\dfrac{2x^3}{3x^2-4}$	o. $f(x)=\dfrac{4x^3}{4x^2+3}$
p. $f(x)=\dfrac{x+1}{2x^2+2x+1}$	q. $f(x)=\dfrac{4x+8}{5x}$	r. $f(x)=\dfrac{5x^2}{x+3}$

3. Find the derivative **using the definition of the derivative** (one of the limit definitions).

Developing		
a. $f(x)=\dfrac{1}{x}$	b. $f(x)=x^2$	c. $f(x)=mx+b$
d. $h(t)=80-4.9t^2$	e. $f(x)=x^3$	f. $f(x)=x+5$
g. $f(x)=2x^2$	h. $f(x)=8x^2-3x+12$	i. $f(x)=\dfrac{1}{x-7}$
Proficient		
s. $g(x)=x^2-\dfrac{1}{x}$	t. $g(x)=\sqrt{3x-1}$	u. $g(x)=\dfrac{1}{\sqrt{x}}$
v. $g(x)=x+\dfrac{1}{x^2}$	w. $g(x)=\dfrac{x}{x+1}$	x. $h(x)=\dfrac{1}{x^2}$

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Extending		
d. $f(x) = \sqrt{169 - x^2}$	e. $f(x) = \frac{2}{\sqrt{2x+1}}$	f. $g(x) = \frac{2x-1}{x+2}$

4. Use the previous examples to find the slope at the point $x = 4$.
5. Use the previous examples to find the equation of the tangent that is perpendicular to the line

$$y = 2x - 5$$