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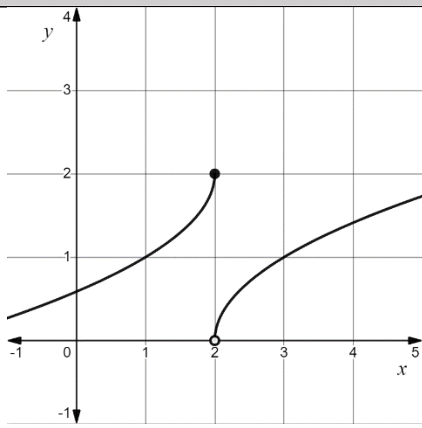
Chapter 2 Review
Limits and Derivatives

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 2.1	Finite limits and continuity.

1. Find the following limits.

Developing



a. $\lim_{x \to 2^-} f(x)$

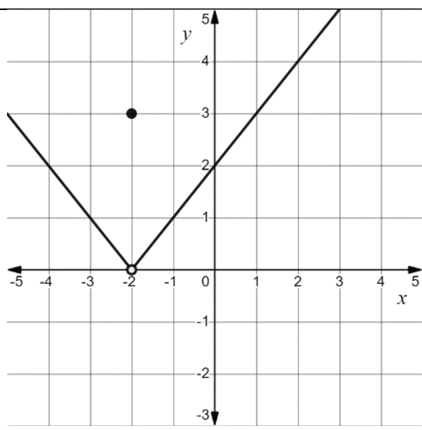
$= 2$

b. $\lim_{x \to 2^+} f(x)$

$= 0$

c. $\lim_{x \to 2} f(x)$

$=$ DNE



d. $\lim_{x \to -2^-} f(x)$

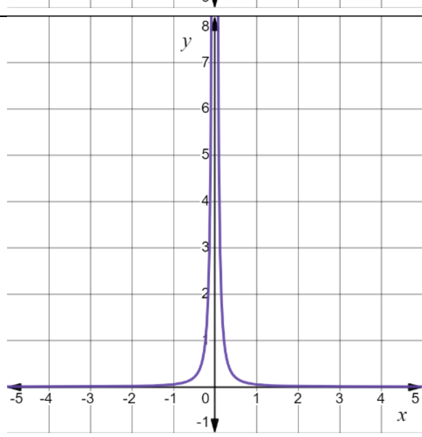
$= 0$

e. $\lim_{x \to -2^+} f(x)$

$= 0$

f. $\lim_{x \to -2} f(x)$

$= 0$



g. $\lim_{x \to 0^-} f(x)$

$= \infty$

h. $\lim_{x \to 0^+} f(x)$

$= \infty$

i. $\lim_{x \to 0} f(x)$

$= \infty$

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j. $\lim_{x\to -2} x^3 + 6x^2 - 16$ $= 0$	k. $\lim_{x\to 4} \frac{x^2+9}{x^2-1}$ $= \frac{5}{3}$	l. $\lim_{x\to 4} \frac{x^2-16}{x^2+x-20}$ $= \frac{8}{9}$
m. $\lim_{x\to 0} \frac{x^2+2x}{x-2x^2}$ $= 2$	n. $\lim_{x\to 1} \frac{1-x^2}{x^2+5x-6}$ $= -\frac{2}{7}$	o. $\lim_{x\to 1} \frac{x^2+x-2}{x^2-4x+3}$ $= -\frac{3}{2}$
p. $\lim_{x\to 3} \frac{x^3-27}{x-3}$ $= 27$	q. $\lim_{x\to 1} \frac{x^3-3x^2+2x}{x-1}$ $= -1$	r. $\lim_{x\to 2} \frac{x^3-4x}{x^3-2x^2}$ $= 2$
s. $\lim_{x\to a} \frac{1/x - 1/a}{x-a}$ $= -\frac{1}{a^2}$	t. $\lim_{x\to 0} \frac{1/(3+x) - 1/3}{x}$ $= -\frac{1}{9}$	u. $\lim_{x\to -a} \frac{x^3+a}{x+a}$ $= 3a^2$
v. $\lim_{x\to 3} \frac{x-3}{x^3-27}$ $= \frac{1}{27}$	w. $\lim_{x\to 2} \frac{1-4/x^2}{1-2/x}$ $= 2$	x. $\lim_{x\to 4^-} \frac{x-4}{\lvert x-4\rvert}$ $= -1$
y. $\lim_{x\to 1} \frac{x-1}{\lvert x-1\rvert}$ $=$ DNE	z. $\lim_{x\to 1} \begin{cases} \frac{1}{x+2}, & x<1 \\ 1-2x, & x>1 \end{cases}$ $=$ DNE	aa. $\lim_{x\to 3} \begin{cases} x^2-1, & x<3 \\ (x-1)^3, & x>3 \end{cases}$ $= 8$
bb. $\lim_{x\to 3} \frac{4x^2-36}{2x-6}$ $= 12$	cc. $\lim_{x\to -1} \frac{x^3+1}{x^4-1}$ $= -\frac{3}{4}$	dd. $\lim_{x\to 2} \frac{2x^2-x-6}{3x^2-7x+2}$ $= \frac{7}{5}$

Proficient

a. $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$ $= \frac{1}{2}$	b. $\lim_{x\to 25} \frac{5-\sqrt{x}}{25-x}$ $= \frac{1}{10}$	c. $\lim_{x\to 9} \frac{9-x}{\sqrt{x}-3}$ $= -6$
d. $\lim_{x\to 0} \frac{(x+3)^3-27}{x}$ $= 27$	e. $\lim_{x\to 0} \frac{x^2}{\sqrt{x^2+12}-\sqrt{12}}$ $= 4\sqrt{3}$	f. $\lim_{x\to 3} \left(\frac{1}{x-3} - \frac{6}{x^2-9}\right)$ $= \frac{1}{6}$
g. $\lim_{x\to 5} \frac{x-5}{\sqrt{x-1}-2}$ $= 4$	h. $\lim_{x\to 2} \frac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x}$ $= -\frac{1}{8}$	i. $\lim_{x\to 16} \frac{4-\sqrt{x}}{x-16}$ $= -\frac{1}{8}$

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j. $\lim_{x\to 2}\frac{x^2-x-2}{\lvert x-2\rvert}$ $= \text{DNE}$	k. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$ $=\frac{1}{2}$	l. $\lim_{x\to 0}\frac{2-\sqrt{4+x}}{x}$ $=-\frac{1}{4}$

Extending

a. $\lim_{x\to 0}\frac{\sin 2x}{4x}$ $=\frac{1}{2}$	b. $\lim_{x\to 0}\frac{\sin x}{x^2-3x}$ $=-3$	c. $\lim_{x\to 0}\frac{\sin x+3x+1}{x}$ $=\text{DNE}$
d. $\lim_{x\to 0}\frac{x\sin x}{\lvert x\rvert}$ $=0$	e. $\lim_{x\to 0}\frac{(x+8)^{1/3}-2}{x}$ $=4$	f. $\lim_{x\to 0}\frac{\sqrt{x+1}-\sqrt{2x+1}}{\sqrt{3x+4}-\sqrt{2x+4}}$ $=-2$
g. $\lim_{x\to 1}\frac{x^{1/6}-1}{x-1}$ $=\frac{1}{6}$	h. $\lim_{x\to 5/2}\frac{\lvert 2x-5\rvert(x+1)}{2x-5}$ $=\text{DNE}$	i. $\lim_{x\to 1}\frac{x^2+\lvert x-1\rvert-1}{\lvert x-1\rvert}$ $=\text{DNE}$
j. $\lim_{x\to 27}\frac{27-x}{x^{1/3}-3}$ $=-27$	k. $\lim_{x\to 4}\frac{\sqrt{x}-2}{\sqrt{x^3}-8}$ $=\frac{1}{12}$	l. $\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}$ $=\frac{1}{12}$
m. $\lim_{x\to 1^+}\frac{1/x-1}{x^2-2x+1}$ $=-\infty$	n. $\lim_{x\to 0^+}\frac{3+x^{-1/2}+x^{-1}}{2+4x^{-1/2}}$ $=\infty$	o. $\lim_{x\to 0^+}(x+5)\left(\frac{1}{2x}+\frac{1}{x+2}\right)$ $=\infty$
p. $\lim_{x\to 2}\frac{x^3-6x-2}{x^3-4x}$ $=\text{DNE}$		

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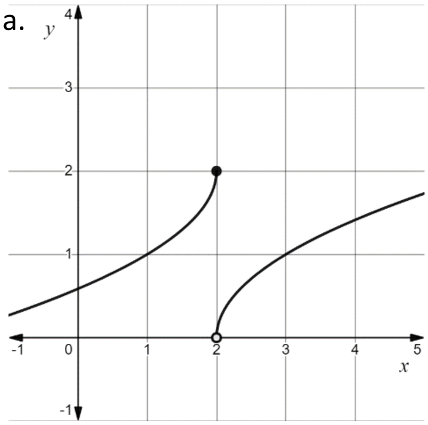
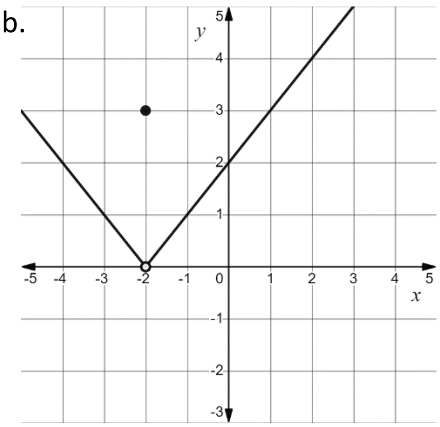
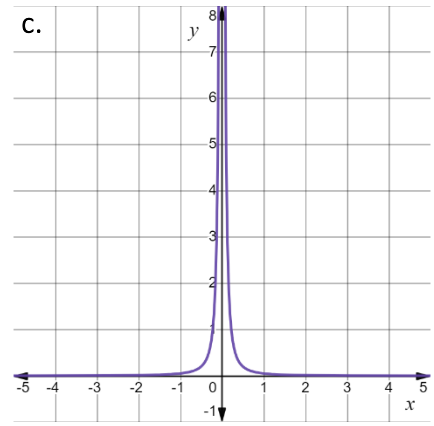
2. Find the point(s) and the associated type(s) of discontinuity.

Developing		
a.  $x = 2$ jump	b.  $x = -2$ point	c.  $x = 0$ vertical asymptote
d. $f(x) = \dfrac{x-1}{x^2+2x-8}$ $x = -4, 2$ vertical asymptotes	e. $f(x) = \dfrac{x^2-16}{x^2+x-20}$ $x = -5$ vertical asymptote $x = 4$ removable	f. $f(x) = \dfrac{x^2+2x}{x-2x^2}$ $x = 0$ removable $x = \dfrac{1}{2}$ vertical asymptote
g. $f(x) = \dfrac{1-x^2}{x^2+5x-6}$ $x = 1$ removable $x = -6$ vertical asymptote	h. $f(x) = \dfrac{x^2+x-2}{x^2-4x+3}$ $x = 3$ vertical asymptote $x = 1$ removable	i. $f(x) = \dfrac{x^3-4x}{x^3-2x^2}$ $x = 2$ removable $x = 0$ vertical asymptote
j. $f(x) = \dfrac{2x^2+5x+20}{x^2+4x}$ $x = -4, 0$ vertical asymptote	k. $f(x) = \dfrac{x^3+1}{x^4-1}$ $x = 1$ vertical asymptote $x = -1$ removable	l. $f(x) = \dfrac{2x^2-x-6}{3x^2-7x+2}$ $x = 2$ removable $x = \dfrac{1}{3}$ vertical asymptote

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Proficient		
a. $f(x) = \frac{x-4}{\lvert x-4 \rvert}$ $x = 4$ jump	b. $f(x) = \begin{cases} \frac{2}{x-1}, & x < 2 \\ x^3 - 2x + 1, & x \geq 2 \end{cases}$ $x = 1$ vertical asymptote $x = 2$ jump	c. $f(x) = \frac{9-x}{\sqrt{x}-3}$ $x = 9$ removable
Extending		
a. $f(x) = \frac{x-3}{x^3-27}$ $x = 3$ removable	b. $f(x) = \frac{x^3-3x-10}{x^3-5x^2-4x+20}$ $x = 2$ vertical asymptote $x = -2, 5$ removable	c. $f(x) = \frac{3x^3-5x^2-4x+4}{3x^3-8x^2+3z+2}$ $x = 2$ removable $x = -\frac{1}{3}, 1$ vertical asymptote

3. Determine constants a and b such that $f(x)$ is continuous for all values of x.

$$f(x) = \begin{cases} ax + 3, & x > 5 \\ 8, & x = 5 \\ x^2 + bx + a, & x < 5 \end{cases}$$

$$a = 1, b = -\frac{18}{5}$$

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Chapter 2 Review
Limits and Derivatives

For each type of question, the achievement level is indicated. Showing work is an important strategy in communicating your knowledge and ideas so please be thorough.

Learning Goal 2.2	Infinite limits and the definition of the derivative.

1. Determine the value of the infinite limit.

Developing		
a. $\lim_{x\to\infty} \frac{2x^2-3x+7}{x^2+47x+1}$ $= 2$	b. $\lim_{x\to\infty} \frac{x^2-16}{x^2+x-20}$ $= 1$	c. $\lim_{x\to\infty} \frac{2x^2+3}{5x^2+x}$ $= \frac{2}{5}$
d. $\lim_{x\to\infty} \frac{1-x^2}{x^2+5x-6}$ $= -1$	e. $\lim_{x\to\infty} \frac{x^2+x-2}{3x^2-4x+3}$ $= \frac{1}{3}$	f. $\lim_{x\to\infty} \frac{x^2-4x}{x^3-2x^2}$ $= 0$
g. $\lim_{x\to\infty} \frac{5x^3-3x^2+1}{x^2+2x+4}$ $= \infty$	h. $\lim_{x\to-\infty} \frac{3x^3+x^2+1}{x^3+1}$ $= 3$	i. $\lim_{x\to\infty} \frac{x^5-x^3+x-1}{x^6+2x^2+1}$ $= 0$
j. $\lim_{x\to-\infty} (2x^3-x)$ $= -\infty$	k. $\lim_{x\to-\infty} \frac{x+2}{x^2+x+1}$ $= 0$	l. $\lim_{x\to-\infty} \frac{3x^3}{3x^2-2}$ $= -\infty$
m. $\lim_{x\to-\infty} \frac{2x^2}{x^2-4}$ $= 2$	n. $\lim_{x\to\infty} -\frac{3x^2}{4x+4}$ $= -\infty$	o. $\lim_{x\to\infty} \frac{2x^3}{3x^2-4}$ $= \infty$
p. $\lim_{x\to-\infty} \frac{4x^3}{4x^2+3}$ $= \infty$	q. $\lim_{x\to\infty} \frac{x+1}{2x^2+2x+1}$ $= 0$	r. $\lim_{x\to-\infty} \frac{\sqrt{2x^2+3}}{2x+3}$ $= -\frac{\sqrt{2}}{2}$
s. $\lim_{x\to-\infty} \frac{\sqrt{2x^2+1}}{4x+2}$ $= -\frac{\sqrt{2}}{4}$	t. $\lim_{x\to-\infty} \frac{4x+8}{5x}$ $= \frac{4}{5}$	u. $\lim_{x\to-\infty} \frac{5x^2}{x+3}$ $= -\infty$
Proficient		
a. $\lim_{x\to\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}}$ $= 1$	b. $\lim_{x\to\infty} \frac{x+5-{}^{2}/_{x}-{}^{1}/_{x^3}}{3x+12-{}^{1}/_{x^2}}$ $= \frac{1}{3}$	c. $\lim_{x\to\infty} \frac{x+x^{1/2}+x^{1/3}}{x^{2/3}+x^{1/4}}$ $= \infty$

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d. $\lim_{x\to\infty} \frac{1-\left(x/_{x-1}\right)}{1-\sqrt{x/_{x-1}}}$ $= 2$	e. $\lim_{x\to-\infty} \frac{x+x^{-1}}{1+\sqrt{1-x}}$ $= -\infty$	f. $\lim_{x\to\infty} \frac{x^{-1}+x^{-1/2}}{x+x^{-1/2}}$ $= 0$
g. $\lim_{x\to\infty} \frac{x+x^{-2}}{2x+x^{-2}}$ $= \frac{1}{2}$	h. $\lim_{x\to\infty} \frac{5+x^{-1}}{1+2x^{-1}}$ $= 5$	i. $\lim_{x\to\infty} \frac{4x}{\sqrt{2x^2+1}}$ $= 2\sqrt{2}$
j. $\lim_{x\to\infty} (x+5)\left(\frac{1}{2x}+\frac{1}{x+2}\right)$ $= \frac{3}{2}$	k. $\lim_{x\to-\infty} \frac{x^4+1}{x^3-1}$ $= -\infty$	l. $\lim_{x\to\infty} \left(\frac{\ln x}{x^4}+1\right)$ $= 1$
m. $\lim_{x\to\infty} (-e^{-3x}-1)$ $= 0$	n. $\lim_{x\to\infty} (e^x-3)$ $= \infty$	o. $\lim_{x\to-\infty} -e^{-4x}$ $= -\infty$
Extending		
a. $\lim_{x\to\infty} \left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$ $= 1$	b. $\lim_{x\to\infty} \frac{1-\sqrt{x/_{x+1}}}{2-\sqrt{4x+1/_{x+2}}}$ $= \frac{2}{7}$	c. $\lim_{x\to\infty} \frac{e^x+x^4}{x^3+5\ln x}$ $= \infty$
If $1 < a < b$ then $f(x) = b^x$ grows faster than $g(x) = a^x$ as $x \to \infty$. Use this idea for the following questions.		
d. $\lim_{x\to\infty} \frac{2^x+5(3^x)}{3(2^x)-3^x}$ $= -5$	e. $\lim_{x\to-\infty} \frac{2^x+5(3^x)}{3(2^x)-3^x}$ $= \frac{1}{3}$	f.

2. Find the equation(s) of any and all asymptotes that exist (vertical, horizontal or slant).

Proficient		
a. $f(x) = \frac{2x^2-3x+7}{x^2+47x+1}$ $y = 2$ horizontal asymptote $x = \frac{-47 \pm 21\sqrt{5}}{2}$ vertical asymptotes	b. $f(x) = \frac{x^2-16}{x^2+x-20}$ $y = 1$ horizontal asymptote $x = -5$ vertical asymptote	c. $f(x) = \frac{2x^2+3}{5x^2+x}$ $y = \frac{2}{5}$ horizontal asymptote $x = -\frac{1}{5}, 0$ vertical asymptotes

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d. $f(x) = \frac{1-x^2}{x^2+5x-6}$ $y = 1$ horizontal asymptote $x = -6$ vertical asymptote	e. $f(x) = \frac{x^2+x-2}{3x^2-4x+3}$ $y = \frac{1}{3}$ horizontal asymptote	f. $f(x) = \frac{x^2-4x}{x^3-2x^2}$ $y = 0$ horizontal asymptote $x = 0, 2$ vertical asymptotes
g. $f(x) = \frac{5x^3-3x^2+1}{x^2+2x+4}$ $y = 5x - 13$ slant asymptote	h. $f(x) = \frac{3x^3+x^2+1}{x^3+1}$ $y = 3$ horizontal asymptote $x = -1$ vertical asymptote	i. $f(x) = \frac{x^5-x^3+x-1}{x^6+2x^2+1}$ $y = 0$ horizontal asymptote
j. $f(x) = \frac{x+2}{x^2+x+1}$ $y = 0$ horizontal asymptote	k. $f(x) = \frac{3x^3}{3x^2-2}$ $y = x$ slant asymptote $x = \pm\frac{\sqrt{6}}{3}$ vertical asymptotes	l. $f(x) = \frac{2x^2}{x^2-4}$ $y = 2$ horizontal asymptote $x = \pm 2$ vertical asymptotes
m. $f(x) = -\frac{3x^2}{4x+4}$ $y = -\frac{3}{4}(x-1)$ slant asymptote $x = -1$ vertical asymptote	n. $f(x) = \frac{2x^3}{3x^2-4}$ $y = \frac{2}{3}x$ slant asymptote $x = \pm\frac{2\sqrt{3}}{3}$ vertical asymptotes	o. $f(x) = \frac{4x^3}{4x^2+3}$ $y = x$ slant asymptote
p. $f(x) = \frac{x+1}{2x^2+2x+1}$ $y = 0$ horizontal asymptote	q. $f(x) = \frac{4x+8}{5x}$ $y = \frac{4}{5}$ horizontal asymptote $x = 0$ vertical asymptote	r. $f(x) = \frac{5x^2}{x+3}$ $y = 5x - 15$ slant asymptote $x = -3$ vertical asymptote

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3. Find the derivative **using the definition of the derivative** (one of the limit definitions).

Developing		
a. $f(x) = \frac{1}{x}$ $f'(x) = -\frac{1}{x^2}$	b. $f(x) = x^2$ $f'(x) = 2x$	c. $f(x) = mx + b$ $f'(x) = m$
d. $h(t) = 80 - 4.9t^2$ $h'(t) = -9.8t$	e. $f(x) = x^3$ $f'(x) = 3x^2$	f. $f(x) = x + 5$ $f'(x) = 1$
g. $f(x) = 2x^2$ $f'(x) = 4x$	h. $f(x) = 8x^2 - 3x + 12$ $f'(x) = 16x - 3$	i. $f(x) = \frac{1}{x-7}$ $f'(x) = -\frac{1}{(x-7)^2}$
Proficient		
s. $g(x) = x^2 - \frac{1}{x}$ $g'(x) = 2x + \frac{1}{x^2}$	t. $g(x) = \sqrt{3x-1}$ $g'(x) = \frac{3}{2\sqrt{3x-1}}$	u. $g(x) = \frac{1}{\sqrt{x}}$ $g'(x) = -\frac{1}{2x^{3/2}}$
v. $g(x) = x + \frac{1}{x^2}$ $g'(x) = 1 - \frac{2}{x^3}$	w. $g(x) = \frac{x}{x+1}$ $g'(x) = \frac{1}{(x+1)^2}$	x. $h(x) = \frac{1}{x^2}$ $h'(x) = -\frac{2}{x^3}$
Extending		
d. $f(x) = \sqrt{169 - x^2}$ $f'(x) = -\frac{x}{\sqrt{169 - x^2}}$	e. $f(x) = \frac{2}{\sqrt{2x+1}}$ $f'(x) = -\frac{2}{(2x+1)^{3/2}}$	f. $g(x) = \frac{2x-1}{x+2}$ $g'(x) = \frac{5}{(x+2)^2}$

4. Use the previous examples to find the slope at the point $x = 4$.
5. Use the previous examples to find the equation of the tangent that is perpendicular to the line

$$y = 2x - 5$$